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A mean approximation based bidimensional empirical mode decomposition with application to image fusion

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ABSTRACT

Empirical mode decomposition (EMD) is an adaptive decomposition method, which is widely used in time-frequency analysis. As a bidimensional extension of EMD, bidimensional empirical mode decomposition (BEMD) presents many useful applications in image processing and computer vision. In this paper, we define the mean points in BEMD 'sifting' processing as centroid point of neighbour extrema points in Delaunay triangulation and propose using mean approximation instead of envelope mean in 'sifting'. The proposed method improves the decomposition result and reduces average computation time of 'sifting' processing. Furthermore, a BEMD-based image fusion approach is presented in this paper. Experimental results show our method can achieve more orthogonal and physical meaningful components and more effective result in image fusion application.

1. Introduction

Empirical mode decomposition (EMD) is a data driven processing algorithm, which employs no predetermined filter. EMD decomposes signal based on the local characteristics scale and presents time-frequency-energy distribution of data, which is suited to analyse nonlinear and nonstationary signals [1]. Due to the data-driven property, EMD has been found many successful applications in variant areas, such as biological, medical sciences, astronomy, engineering and others [12]. Recently, EMD based data-driven sparse representation over a highly redundant time-frequency dictionary method [3] has been proposed and used to detect intrinsic pattern or physical information from data, where traditional methods decompose signal to artificial harmonics components.

For two-dimensional signals, bidimensional EMD (BEMD) is developed as a two-dimensional version of EMD. There are many applications of EMD/BEMD reported in the literatures for image analysis and processing. Some different approaches to extend EMD to two-dimensional are reported [2,12,16]. EMD/BEMD is widely used in image compression [6], multi-media coding [23], face preprocessing [24], image fusion [26], etc.

Ensemble EMD (EEMD) [15], which was firstly proposed by Wu et al. to reduce the mode mixing problem, has been extended to multi-dimensional ensemble EMD (M-EEMD) [16]. Bidimensional statistical empirical mode decomposition (BSEMD) [28] termed a smoothing procedure instead of 2D interpolation by constructing envelopes coupling with a new extrema identification method. Dragomiretskiy et al. [18,19] created an entirely non-recursive 2D variational mode decomposition (VMD) model, where the modes are detected concurrently. VMD decomposed an image into a given number of modes such that each individual mode has limited bandwidth [19].

'Sifting' process is the main procedure in EMD/BEMD, which generates the upper and lower envelope by interpolation method. In 2D condition, envelope generation methods require high computation time with interpolation approaches, such as radial basis function [4] or thin-plate splines [6]. Some other envelope generation algorithms are also proposed, such as finite elements based on Delaunay triangulation [13], spatial domain order-statistics filter [11], oblique-extrema-based approach [22]. These approaches have improved computation speed and decomposition result of EMD/BEMD, but more efficient and effective methods still need further research.

In this paper, we proposed an effective and efficient BEMD method. The proposed method utilizes a mean approximation based generation approach. By defining the mean points in two dimensions, we build a mean approximation instead of envelope mean by Delaunay triangular mesh. Our experiments show that the proposed method decomposes images to more orthogonal components and consumes less computational time. In the following, we will introduce the basic concept and algorithm of BEMD in Section 2. Section 3 describes the proposed mean approximation based BEMD method. Some decomposition result and discussion

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are shown in Section 4. Further, a fusion image method based on the proposed mean approximation based BEMD method is described in Section 5. Section 6 gives a conclusion of this paper.

2. Overview of bidimensional empirical mode decomposition

Nonstationary signals have statistical properties that vary as a function of time and should be analysed differently from stationary data. Rather than assuming that a signal is a linear combination of predetermined basis functions, in EMD, the data is instead thought of as a superposition of fast oscillations onto slow oscillations [14]. EMD is designed to decompose those oscillations which intrinsically present in the signal.

EMD decomposes signal into components called Intrinsic Mode Functions (IMFs) satisfying the following two conditions [1]: (a) The numbers of extrema and zero-crossings must be either equal or differ at most by one; (b) At any point, the mean value of the envelope defined by the local maxima and the envelope by the local minima is zero.

Huang [1] also proposed an algorithm called 'sifting' to extract IMFs $J_k(t)$ from the original signal $f(t)$:

$$f(t) = \sum_{k=1}^{K} J_k(t) + r_K(t) \tag{1}$$

where $J_k(t)$, $k = 1, \dots, K$ is IMFs and r_K is the residue.

The EMD is originally developed for one-dimensional (1D) data. Nunes [4] firstly extended it to two-dimensional BEMD, and decomposed images to bidimensional IMFs (BIMFs). The Bidimensional EMD (BEMD) process is conceptually the same as the one dimension EMD. The main process of the BEMD can be described as:

Step 1 Find the location of local extrema (both maxima and minima) of the image $I(i, j)$;

Step 2 Generate envelopes by maximum points (respectively, minima points) using 2D interpolation methods. Compute the local mean m as the average of upper and lower envelope;

Step 3 Subtract the mean from the image to obtain a proto-BIMF $r = I - m$ and judge whether r is a BIMF. If it is, go to Step 4. Otherwise, repeat Step 1 and Step 2 using the proto-BIMF r as input, until the latest proto-BIMF turns to be a BIMF;

Huang proposed the Cauchy standard deviation (SD) criterion to judge a BIMF in his original paper [1], for the two-dimensional case, the SD criterion is:

$$SD = \sum_{i=1}^{M} \sum_{j=1}^{N} \frac{(r_k(i, j) - r_{k-1}(i, j))^2}{r_{k-1}^2(i, j)} \tag{2}$$

if SD is larger than a threshold ε, repeat the step 1 to step 3 with $r_k(i, j)$ as the input, otherwise, $r_k(i, j)$ is a BIMF $d_k(i, j)$. Follow the method in Nunes [5], in this paper, $\varepsilon = 0.12$ is used as the SD criterion;

Step 4 Input the proto-BIMF r to the loop from Step 1 to Step 3 to get the next remained BIMF until it cannot be decomposed further.

After the BEMD, the decomposition of the image can be rewritten as following form:

$$I(i, j) = \sum_{k=1}^{K} d_k(i, j) + r_K(i, j) \tag{3}$$

The $d_k(i, j)$ is the kth BIMF of the images, and $r_K(i, j)$ is the residual function. The basic idea of the BEMD is to decompose a signal into the sum of these BIMFs and the residue function, so that d_1 represents the highest frequency of I, d_2 represents the second highest frequency of I and r_K represents the lowest frequency of I. The principle of BEMD is to extract variations from the data by separating the fluctuation from the mean [10].

3. Proposed mean approximation based BEMD

In our previous work [20], to reduce the overshooting problem of cubic interpolation in one dimensional EMD, we proposed using mean points to build the extrema mean approximation instead of building average of upper and lower envelope. For two-dimensional signal images, mean approximation is built with careful mean point definition in this paper and we further discuss the mean approximation generating method for two-dimensional in this section.

The first step in 'sifting' is to find the local extrema points in image. Following the method in [11], neighbour location method is used to detect the local maxima or minima points. A point $u(i, j)$ is considered as a local maximum (or local minimum) if its value is strictly larger (or lower) than the value of u at the nearest 8-neighbours of points (i, j) [11].

3.1. Mean approximation in two-dimensional signal

In previous BEMD methods, RBF based [5] or Delaunay based method [7] etc. is used to build the upper and lower envelopes from the maximum and minimum points. These methods, which first generate upper and lower envelopes and then compute the mean of these envelopes, can be denoted as 'envelope mean' methods. Different from these 'envelope mean' methods, we propose a 'mean approximation' method in this section.

Definition 1 *(Mean approximation).* Mean approximation is an approximation surface generated from mean points of extrema points.

Definition 2 *(Mean point).* Mean point is the centroid point of a neighbour extrema points set. In $K - 1$ dimensional space, the K neighbour points are these points which one point as a starting point and other $K - 1$ points as end points, formed a set of linear independent vectors.

Set the extrema points are $\{u_1, u_2, \dots, u_K\} \in \mathbb{R}^{K-1}$, from one point u_k to other $K - 1$ points form a set of vectors $\{\overrightarrow{u_k u_1}, \overrightarrow{u_k u_2}, \dots, \overrightarrow{u_k u_K}\}$. These vectors are linearly independent if there exist scalars $a_1, a_2, \dots, a_{K-1}$ not all zero, such that

$$a_1 \overrightarrow{u_k u_1} + a_2 \overrightarrow{u_k u_2} + \cdots + a_{K-1} \overrightarrow{u_k u_K} = 0 \tag{4}$$

These extrema points $\{u_1, u_2, \dots, u_K\} \in \mathbb{R}^{K-1}$ form a neighbour extrema points set. The centroid point of the finite set of K neighbour extrema points $\{u_1, u_2, \dots, u_K\} \in \mathbb{R}^{K-1}$ is

$$u_{mean} = \frac{1}{K} \sum_{i=1}^{K} u_i \tag{5}$$

In two-dimensional, mean point is defined as a centroid of the triangle which triangle vertices are extrema points built by Delaunay triangular mesh in our discussion.

In our method, the all extrema points (maximum or minimum) treated as the same. A point $u \in \mathbb{R}^2$ is called extrema point if it is either maximum point or minimum point. As illustrated in Fig. 1, we divide the spatial domain into triangular meshes which vertices are the extrema points. This can be built by Delaunay triangulation method [7,13]. In such a triangular mesh, any triangle does not overlap with any other triangles in the mesh and a vertex of a triangle is not in the interior of an edge of another triangle in the mesh [13]. As in [13], symmetric extension is employed to mirror the image boundary parts to extended parts. For more discussion about the triangular mesh by Delaunay method, one can find it in [7,9,13].