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DOA estimation in conformal arrays based on the nested array principles

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ABSTRACT

The nested array structure has attracted great attention recently due to its ability in reducing the number of sensors in an array and at the same time preserving the array performance. While a uniform linear array (ULA) can detect at most $N-1$ sources with N sensors, a nested array can provide $O(N^2)$ degrees of freedom with the same number of sensors; allowing us to detect K sources with $K > N$ sensors. Direction of arrival (DOA) estimation in a conformal array is a challenging task. In this article, by breaking the conformal array into smaller sub-arrays and using an interpolation technique, we employ the nested array principles to detect more number of sources than sensors. This comes at the cost of more snapshots and lower resolution, in the DOA estimation of an arbitrarily-shaped conformal array. Each sub-array in the conformal array is selected such that the "shadow effect" which leads to an incomplete steering vector in the DOA estimation algorithm is eliminated. The selected sub-arrays are then transformed to virtual nested arrays where more degrees of freedom can be obtained by applying the MUSIC algorithm for DOA estimation. The application of our proposed method is highlighted by considering a set of comprehensive examples for cylindrical and spherical arrays.

1. Introduction

A conformal antenna array is an array with its antennas conformed to its bearing surface. Due to this flexibility, conformal arrays have found great applications in many modern defense and wireless communication systems [1,2]. Unfortunately, the existing beamforming and direction-of-arrival (DOA) estimation algorithms which have been developed for conventional planar arrays cannot be directly applied to conformal arrays since these algorithms assume elements with identical patterns which is not valid in conformal arrays. Due to the curvature of the bearing surface, each element in a conformal array has a far-field contribution in the direction of the incident signals which is different from the contributions of other elements. This means that, unlike conventional planar arrays, one cannot separate the element pattern from the array factor in the analysis of a conformal array [1]. To use high resolution DOA estimation algorithms such as MUSIC [3] or ESPRIT [4] for arrays with arbitrary geometries, interpolation techniques have been employed by some authors [5–13]. The interpolation technique is a method which enables us to map an array with M sensors to an array with $N \geq M$ virtual sensors. The manifold matrix of a non-uniform planar array is, in general, not Vandermonde. By using array interpolation, a virtual uniform planar array is generated which ensures obtaining a Vandermonde matrix. Particularly, in [13] a DOA estimation method for conformal arrays is proposed which uses the interpolation technique to transform a conformal array into a uniform linear array (or a uniform planar array). By applying LS-ESPRIT, 2D-DFT ESPRIT and MUSIC algorithms to the interpolated array the authors in [13] have shown that this method can identify the DOA of different sources with great accuracy for different conformal array geometries. Array interpolation has also been applied to both narrowband and wideband signals [5–11].

The number of signals that can be detected by conventional arrays is always less than the number of array sensors, e.g. an array with N sensors can resolve at most $N-1$ sources [14]. However, in many applications, while the size of the array is limited, it is required to detect more number of signals than the number of elements. Such examples can be found, for example, in Unmanned Aerial Vehicles (UAV). UAVs are highly mobile units that can be employed as safe and efficient communication relays for various surveillance, reconnaissance, and other tactical missions [15]. While increasing the degrees of freedom (DOF) in the array of a UAV can increase its performance, the physical size and the weight of the array is a limiting factor.

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Different methods have been proposed so far in order to increase the DOF of conventional arrays [16–20]. A large number of such methods use the sparse array structure [16–20]. A sparse array is a uniform array where certain fractions of its elements have been removed randomly. A sparse array has a large variation in its peak sidelobe level. Hence, optimization techniques must be used to remove appropriate elements to have a good sidelobe behavior [16–18]. These sparse arrays, also called thinned arrays, are designed under specific considerations such as a particular radiation pattern or impedance matching of the array elements [19,20].

Minimum redundancy arrays (MRA) constitute a sub-group of sparse arrays, which use the idea of co-array design to increase the DOF [21–24]. Co-array theory is based on the fact that the difference of element positions in an array structure appears in the signal correlation matrix [25]. MRAs provide more DOF both by reducing the number of elements and by manipulating the received signals in order to extract the missing data of the removed elements. The main drawback of the minimum redundancy arrays is that they do not have a closed form expression for the array geometry and the achievable DOF.

Another group of sparse arrays, the so-called co-prime array, was proposed and developed in [26,27], using $N_{c1} + N_{c2}$ sensors to obtain $O(N_{c1}N_{c2})$ degrees of freedom for DOA estimation where N_{c1} and N_{c2} are co-prime numbers. In co-prime arrays, two separate ULAs with element spacing dN_{c1} and dN_{c2} are placed together on a row where the difference of the element positions forms a uniform spatial sampling with the aid of the co-array theory.

The newly proposed structure, namely the nested array, can overcome the issues of the MRAs and at the same time can achieve $O(N^2)$ degrees of freedom using N sensors by combining two or more Uniform Linear Arrays (ULA) with increasing inter-element spacing [28]. Furthermore, compared to a co-prime array, a nested array requires less number of sensors to achieve the same DOF. By using the co-array theory, a nested array can achieve more DOF but it is at the cost of lower detection resolution compared to conventional methods with the same number of snapshots because the finite sample size of the signals can have more negative impact on the accuracy of the covariance matrix in a nested array than a conventional array. This is an inherent problem of nested arrays and has also been reported in previous literature [28,29].

This issue has been mitigated recently by exploiting the numerous iterations of subsets of the whole data set in both nested arrays and co-prime arrays [30]. Due to these prominent features, the nested array structure has been recently extended from one-dimensional arrays to two-dimensional arrays [29,31] and from narrowband sources to wideband signals [32].

Our aim in this article is to increase the DOF in the DOA estimation of a conformal array to detect more number of sources than the sensors, thereby improving the performance of a conformal array. For this purpose, we map the given conformal array to a corresponding linear or planar nested array using the interpolation technique. First, the desired spatial angle in the conformal array is divided into small sectors to reduce the interpolation error and eliminate the shadow effect. The shadow effect appears when some of the elements of the conformal array are not able to receive the incoming signals due to the curvature of the arrays. Each sector is then transformed, by means of an appropriate transformation matrix, into a corresponding virtual nested array. This transformation is also applied to the impinging signals on the conformal array. Next the conventional high resolution DOA estimation algorithms can be applied to each virtual nested array to provide more DOF. Selecting the size of each sub-array and the number of elements in its corresponding nested array are design parameters that determine the accuracy of the DOA estimation. In summary, our approach includes two steps: a) a transformation from conformal geometry to a virtual linear (planar) array geometry, b) increasing the DOF by applying the nested array principle to this virtual linear (planar) array.

The organization of the paper is as follows. The next section briefly reviews the nested array principles and its associated spatial smoothing technique and shows how this structure can increase the DOF in both 1D (linear) and 2D (planar) arrays. In section 3, the "shadow effect" problem in a conformal array is discussed. Section 4 presents our approach to develop the nested array structure to a conformal array and shows how the detected number of sources can be increased in the DOA estimation based on this new design. Simulation results are presented in Section 5 for a cylindrical conformal array as a typical example for the 1D case and a spherical conformal array as a typical example for the 2D case. Finally, Section 6 concludes the paper.

2. Nested array principles

Nested arrays are a subgroup of sparse arrays and use the co-array theory to increase the DOF. Since the distribution of the sensors in a nested array is non-uniform, we need to first consider the signal model in a non-uniform 1D (linear) array.

We will use bold lowercase and uppercase characters to denote vectors and matrices, respectively. The symbol $(.)^*$ denotes the conjugate operator, $(.)^{\mathrm{T}}$ the transpose operator, and $(.)^{\mathrm{H}}$ the conjugate transpose operator. N is used for the number of sensors in a nested array and M is used for the number of sensors in a conformal array. Since in our case, the nested array is a virtual array whereas the conformal array is the actual (physical) array we use two different symbols for referring to their sensors.

2.1. Non-uniform linear array

Consider an N element non-uniform linear antenna array. Let $\mathbf{a}(\theta)$ denote the $N \times 1$ steering vector corresponding to the direction θ defined as [28],

$$\mathbf{a}(\theta) = \left[e^{j(\frac{2\pi}{\lambda})d_1 \sin\theta} \; e^{j(\frac{2\pi}{\lambda})d_2 \sin\theta} \dots e^{j(\frac{2\pi}{\lambda})d_N \sin\theta}\right]^{\mathrm{T}} \tag{1}$$

where d_i denotes the position of the ith sensor and λ is the signal wavelength. Suppose that D narrowband sources are impinging on this array from directions $\{\theta_d, d = 1, 2, \dots, D\}$ with powers $\{\sigma_d^2, d = 1, 2, \dots, D\}$, respectively. We can write the received signal model as [28],

$$\mathbf{y}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{e}(t) \tag{2}$$

where $\mathbf{y}(t) = [y_1(t) \; y_2(t) \dots y_N(t)]^{\mathrm{T}}$ is the vector of the received signals, $\mathbf{A} = [\mathbf{a}(\theta_1) \; \mathbf{a}(\theta_2) \dots \mathbf{a}(\theta_D)]$ is an $N \times D$ matrix which represents the array manifold, and $\mathbf{x}(t) = [s_1(t) \; s_2(t) \dots s_D(t)]^{\mathrm{T}}$ denotes the source signal vector. The impinging signals are assumed to be uncorrelated with each other. The channel noise $\mathbf{e}(t)$ is assumed to be temporally and spatially white and uncorrelated from the source with the covariance of $\sigma_e^2\mathbf{I}$. From (2), the autocorrelation matrix $\mathbf{R_{yy}} = E[\mathbf{y}\mathbf{y}^{\mathrm{H}}]$ of the received signal is written as [28],

$$\mathbf{R_{yy}} = \mathbf{A}\mathbf{R_{xx}}\mathbf{A}^{\mathrm{H}} + \sigma_e^2\mathbf{I} = \mathbf{A}\mathbf{\Lambda}\mathbf{A}^{\mathrm{H}} + \sigma_e^2\mathbf{I} \tag{3}$$

where $\mathbf{R_{xx}}$ represents the covariance matrix of the sources, $\mathbf{\Lambda}$ is a diagonal matrix with signal powers at its diagonal elements and $\mathbf{I}$ denotes the identity matrix.

2.2. 1D nested array structure

A simple 1D nested array is a non-uniform linear array consisting of two concatenated ULAs as it is shown in Fig. 1 [28]. With N sensors, the first ULA has N_1 sensors with interelement spacing $d_1 = \lambda/2$ and the second ULA has N_2 sensors with interelement spacing $d_2 = (N_1 + 1)d_1$ [28].