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Multiplicative noise removal via adaptive learned dictionaries and TV regularization

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ABSTRACT

Multiplicative noise removal is a key issue in image processing problem. While a large amount of literature on this subject are total variation (TV)-based and wavelet-based methods, recently sparse representation of images has shown to be efficient approach for image restoration. TV regularization is efficient to restore cartoon images while dictionaries are well adapted to textures and some tricky structures. Following this idea, in this paper, we propose an approach that combines the advantages of sparse representation over dictionary learning and TV regularization method. The method is proposed to solve multiplicative noise removal problem by minimizing the energy functional, which is composed of the data-fidelity term, a sparse representation prior over adaptive learned dictionaries, and TV regularization term. The optimization problem can be efficiently solved by the split Bregman algorithm. Experimental results validate that the proposed model has a superior performance than many recent methods, in terms of peak signal-to-noise ratio, mean absolute-deviation error, mean structure similarity, and subjective visual quality.

1. Introduction

Image denoising is a fundamental problem in both image processing and computer vision with numerous applications. In recent years, the issue of multiplicative noise removal has attracted much attention. Multiplicative noise arises in many imaging applications, such as ultrasound imaging and synthetic aperture radar (SAR) images [1–3]. In this paper, we focus on the multiplicative noise that follows the Gamma distribution. Mathematically, such an image degradation process could be modeled by

$$f = un \tag{1}$$

where $f : \Omega \to R$ represents the contaminated image, $u : \Omega \to R$ is the original clean image and n denotes multiplicative noise, which follows the Gamma distribution with the probability density function

$$pdf(n) = \frac{M^M n^{M-1}}{\Gamma(M)} e^{-Mn} \quad (n \geq 0) \tag{2}$$

where M is an integer related to the noise level, and $\Gamma(\cdot)$ is the Gamma function. The main objective of denoising is to estimate the original image u from an observed noisy image f without losing significant features such as edges and textures. However, multiplicative noise is signal independent and spatially independent. Therefore, multiplicative noise removal becomes a very challenging problem.

To deal with this more complex ill-posed inverse problem, a lot of methods have been proposed. A widely-used regularization term in image processing is total variation (TV) [4–8]. TV-based methods can remove noise in flat region effectively. Furthermore, these methods have shown good performance on edge-preserving. However, they are also known to over-smooth textures, which may cause the loss of fine details. Other widely-used approaches are wavelet-based methods [9–13] and nonlocal-based methods [14–17]. Wavelet-based methods assumed that natural images have sparse representation under some transformation space. These approaches better keep texture features than TV-based methods. However, they are based on a fixed dictionary, which has nothing with the image content. As a result, these approaches may fail to capture the specific characteristics of the processed image, which limits their performances. While nonlocal-based methods take advantage of the similarity of the image patches. Low similarity or dissimilarity of the image patches also limits their performances. In order to overcome these limitations and better consider the distinctive structure of the processed image, sparse representation methods over dictionary learning have been proposed and widely adopted in image processing in the past few years.

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Sparse and redundant representations over dictionary learning have been extensively studied, and significant progress has been made in image processing. This is due to the fact that natural images contain repeated patterns, such as flat region and texture region. Therefore, they can be well approximated as linear combinations of only a few atoms from a dictionary [18–21]. Dictionary learning in image denoising was proposed by Elad and Aharon. They presented the method via sparse and redundant representations over learned dictionaries, called K-SVD [18], which was performed to remove additive Gaussian noise. K-SVD algorithm includes sparse coding stage and a process of updating the dictionary atoms to better fit the data. It first learns a dictionary from the noisy image patches, and then recovers each image patch by using the linear combinations of a few atoms in the learned dictionary. This method provides the state-of-the-art results, and it has been generalized to handle image sequence denoising, deblurring, decomposition, reducing artifacts [22–28]. For example, Zhao and Yang [28] proposed a hyperspectral image (HSI) denoising method by jointly utilizing sparse representation and low-rank constraint. Both the global and local redundancy and correlation (RAC) in spatial/spectral domains were considered. To avoid spectral distortion, the low rank of HSI was incorporated as an additional regularization term. This denoising method for HSI achieves competitive performance.

In this paper, we propose a new model for multiplicative noise removal, which combines the maximum a posteriori (MAP) formulation, the sparse and redundant representations via learned dictionaries and total variation. TV regularization methods can not only remove noise in flat region effectively, but also can reduce the artifacts in smooth regions sometimes caused by patch-based prior of dictionary learning. So our approach combines the advantages of sparse representations over learned dictionaries and the advantages of TV-based methods. Moreover, the proposed model is more universal for various kinds of images. At last, split Bregman algorithm used to solve the optimization problem.

The rest of this paper is organized as follows. The prior works for multiplicative noise removal was briefly reviewed in Section 2. In Section 3, we propose a new model. Experimental results are given to demonstrate the efficiency of the proposed method in Section 4. Finally, conclusions are given in Section 5.

2. Review of prior works for multiplicative noise removal

In this section, we review prior works for multiplicative noise removal briefly. The issue of image denoising is to estimate the original image u from an observed noisy image f. The problem of multiplicative noise removal is developed on the basis of researches on additive noise removal. In view of additive noise removal, the general nonlinear image variation model is

$$\hat{u} = \arg\min_u \left\{ \frac{1}{2} \int_\Omega (u-f)^2 + \lambda \int_\Omega \varphi\big(|\nabla u|\big) \right\} \tag{3}$$

where the second term is regularization term to demand the smooth degree of the estimated image, and λ is the penalty parameter. $\varphi(|\nabla u|) = |\nabla u|$ is TV regularization term and $\varphi(|\nabla u|) = \mu^2 \log(1 + |\nabla u|^2/\mu^2)$ is PM regularization term [29]. Until now, several multiplicative denoising models based on the TV regularization have been proposed. For example, according to MAP analysis, Aubert and Aujol proposed a functional to remove multiplicative noise [6]. In their approach, the Gamma noise with one mean is considered. However, because of the non-convexity, the solution of their model is not necessary to be an optimal solution. In addition, the solution seriously depends on the variable initialization. In [7], Shi and Osher considered a noisy observation $\log f = \log u + \log \eta$ by using the logarithmic transformation, which converted multiplicative noise to additive noise. They considered seeking $\omega = \log u$ based on the noisy observation $\log f$. Their model is strictly convex. Moreover, the convergence of the multiplicative noise model was demonstrated, as well as its regularization effect and its relation to the Bregman distance. Durand et al. [12] introduced an efficient hybrid method (DFN) by using L1 fidelity on frame coefficients in the log-image domain. They used curvelet as the frame, and a Douglas–Rachford splitting algorithm (DRSM) [30] to minimize the energy function. The numerical experiments demonstrated the superior performance to the other compared approaches. Moreover, Teuber et al. [17] considered using a nonlocal filter (NL) to remove multiplicative noise. Some good properties were revealed in the denoising results.

Recently, Huang et al. [31] proposed a new model for multiplicative noise removal. Firstly, they learned a dictionary D from the logarithmic transformed image. For the image patches $R_{ij} \log f$, they seek a dictionary D and sparse coefficients α_{ij} to minimize the cost function

$$\{D, \alpha_{ij}\} = \arg\min_{\{D,\alpha_{ij}\}} \frac{1}{2} \sum_{ij} \|R_{ij} \log f - D\alpha_{ij}\|^2 + \sum_{ij} \mu_{ij} \|\alpha_{ij}\|_0 \tag{4}$$

where R_{ij} is a matrix that extracts the (i, j) block from the image. The hidden parameter μ_{ij} balances sparsity and representation error, which is determined by the sparse coding procedure described in [19]. Secondly, they use it in a variation model built for noise removal. To address the non-convex optimization problem, they introduced the transformation $u = \exp(y)$ and obtain a convex model with respect to the variable y

$$\arg\min_y \lambda\langle y + fe^{-y}, 1_\Omega\rangle + \frac{1}{2} \sum_{ij} \|R_{ij} y - D\alpha_{ij}\|^2 + \gamma \|y\|_{TV} \tag{5}$$

where λ, γ are positive regularization parameters. The first term is data-fidelity term; the second term assume that the small patches of y can be represented sparsely over a certain adaptive dictionary D; the $\|\cdot\|_{TV}$ term is defined by summing over the norm of ∇y, which is efficient to remove noise in cartoon images. They designed an efficient dual approach to solve the model (5). Experimental results show that the algorithm can obtain superior performances than DFN [12], NL [17] and DRSM [30].

3. Our approach for multiplicative noise removal

3.1. Proposed model

As pointed in Section 1, our goal is to estimate the original image u from the observed image f. A classical statistical approach is the MAP estimator

$$\hat{u} = \arg\max_u P(u|f) \tag{6}$$

From Bayes rule, we have $P(u|f) = \frac{P(f|u)P(u)}{P(f)}$. Maximizing $P(u|f)$ amounts to minimizing the log-likelihood

$$-\log\big(P(u|f)\big) = -\log\big(P(f|u)\big) - \log\big(P(u)\big) + \log\big(P(f)\big) \tag{7}$$

The above computation leads to the following functional for restoration images corrupted with Gamma noise

$$\int_\Omega \left(\log u + \frac{f}{u}\right) + \frac{\gamma}{Z} \int_\Omega \phi(u) \tag{8}$$

where the first term is called data-fidelity term, and we assume that u follows a Gibbs prior $g(u) = \frac{1}{Z} e^{-\gamma\phi(u)}$ with a normalizing constant and a non negative given function ϕ. Here we derive the