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Complex bandpass sampling and direct downconversion of multiband analytic signals

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ABSTRACT

The kernel concept of the software defined radio architecture is to eliminate the analogue mixers and to place analogue-to-digital converters as near the antenna as possible. Bandpass sampling can be used for direct downconversion without analogue mixers. In this paper, we report an efficient method to find the ranges of valid bandpass sampling frequency for direct downconverting multiple bandpass analytic signals (single-sideband RF signals). The algorithm results in the ranges of valid bandpass sampling frequency for the complex signals in terms of bandwidths and band positions of the single-sideband RF signals. Compared to real bandpass sampling, the valid sampling frequency ranges are easier to find and the ranges thus obtained having much wider interval than those of real sampling. As a consequence, the complex sampling scheme is more flexible in choosing sampling frequency and more robust to the sampling frequency variation. Furthermore, if spectral inversion is not permitted, then in some cases there will have no applicable sampling frequency under Nyquist rate for real sampling.

1. Introduction

With the advent of faster and less expensive digital hardware, more of the traditionally analogue functions of a radio receiver will be replaced with software or digital hardware. This stimulated the concept of software radio [1]. The goal for software radio is to directly digitize the RF signal at the output of the receiving antenna and therefore implement all receiver functions in either digital hardware or software.

For an analogue signal which has the highest frequency at f_H will need a sampling rate higher than $2f_H$ for aliasing free. This scheme is well known as Nyquist sampling [2]. However, for most radio applications, the required sampling rate would be impractically high if Nyquist sampling is employed. Fortunately, sampling at rates lower than $2f_H$ still can allow for an exact reconstruction of the information content of the analogue signal if the signal is a bandpass signal. An ideal bandpass signal has no frequency components below a low frequency f_L and above a high frequency f_H. We can use a technique known as bandpass sampling to sample a continuous bandpass signal that is with some centre frequency other than 0 Hz. If a bandpass signal is located between f_L Hz and f_H Hz and the bandwidth B is equal to $f_H - f_L$, the classic bandpass theorem for uniform sampling states that the signal can be reconstructed if the sampling rate is at least $f_{s(min)} = 2f_H/n$, where n is the largest integer within f_H/B, denoted by $n = \lfloor f_H/B \rfloor$ [3]. The theoretical minimum bandpass sampling frequency is two times the bandwidth of the bandpass signal. As a consequence, we are more concerned with signal's bandwidth than its highest frequency component.

The principles of real bandpass sampling for a single real RF signal was discussed thoroughly in literature [3] and for two or more real RF signals was discussed in [4–6]. A testing method to determine the bandpass sampling frequency for multiple nonadjacent RF signal is proposed in [4]. In which some constraints on the sampling frequency for avoiding aliasing were developed. Through

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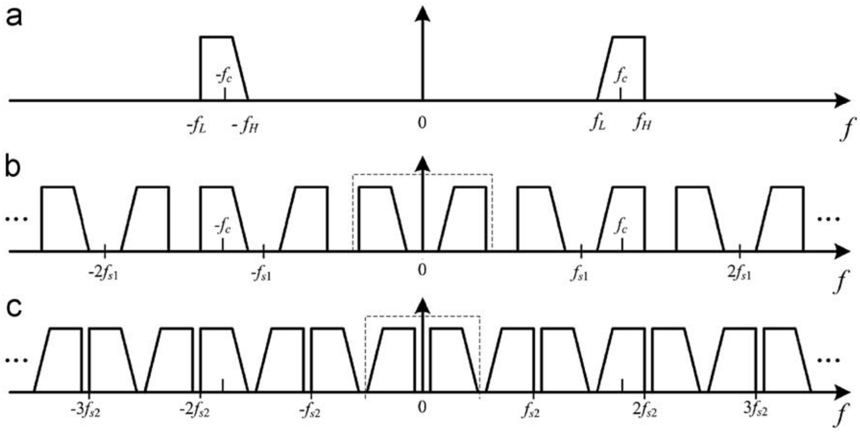

Fig. 1. The spectra of: (a) the original, (b) the sampled signal by sampling frequency f_{s1} and (c) the sampled signal by sampling frequency f_{s2}.

the exhaustive test on the constraints for all frequencies up to the Nyquist rate to find the valid sampling frequency would be required. However, this method can just judge the frequency under tested is an available one or not, rather than to find all the valid frequency ranges. The method in [5] depicts the valid sampling frequency ranges for each RF signal with a graph, and by overlapping the graphs for all RF signals; one can obtain the intersection of the graphs. The frequency within the ranges of the intersection is the valid sampling frequency that suitable for all RF signals. However this method is still impractical in the situations of large number of RF signals or when the ratio of f_H/B is large. Since in these situations it is too hard to find the ranges of intersection or there may have too many intersections to be recognized. In [6], Tseng and Chou proposed an algorithm that is through analyzing the relations between the boundary frequencies of each RF signal to obtain formulas for the ranges of valid bandpass sampling frequency.

In this paper, we focus on the sampling about single-sideband RF signals, which is the issue of complex bandpass sampling (or analytic sampling [7]). In real bandpass sampling, not all the sampling frequency between the minimum frequency ($f_{s(min)} = 2f_H/n$) and Nyquist frequency ($2f_H$) is allowed. This because the aliasing occurred at the negative frequency part overlapped its positive frequency part. But, if sampling applied to the single-sideband signal, the complex bandpass sampled signal did not generate alias if the sampling frequency is larger than signal's bandwidth (f_H–f_L). Hence, the constraints of real sampling can be easily relaxed. Consequently, the method presented in [6] can be much simplified.

2. Bandpass sampling and frequency shifting of a single RF signal

2.1. Real sampling

The conditions for valid uniform bandpass sampling rate of a single RF signals is given by [3]

$$\frac{2f_H}{n} \le f_s \le \frac{2f_L}{n-1} \tag{1}$$

where n is an arbitrary positive integer ensuring that $f_s \ge 2B$.

Consider sampling an RF signal whose spectrum is shown in Fig. 1(a). If the sampling frequency f_s is carefully chosen then no spectrum overlapping occurs. Fig. 1(b) and (c) give the sampled spectrum under two valid sampling frequencies, f_{s1} and f_{s2}. Notice that for sampling frequencies f_{s2}, it provides a sampled baseband spectrum that is inverted from the original positive and negative spectral shapes. This spectral inversion happens whenever n, in (1), is an even integer.

As shown in Fig. 1(b) and (c), the spectrum of the sampled signal can be obtained by replicating the spectrum of the original signal at multiples of f_s. Consequently, a bandpass sampled RF signal which is originally centred at f_c will be successively replicated at the frequencies $f_c^{(m)}$. The $f_c^{(m)}$ is related to f_c and f_s as follows:

$$f_c^{(m)} = \begin{cases} mf_s + \text{rem}(f_c,f_s) & \text{for } \left\lfloor \dfrac{2f_c}{f_s} \right\rfloor \text{ is even} \\ (m+1)f_s - \text{rem}(f_c,f_s) & \text{for } \left\lfloor \dfrac{2f_c}{f_s} \right\rfloor \text{ is odd} \end{cases}, \quad m = 0,1,\ldots,\infty \tag{2}$$

where rem(a, b) denotes the remainder of a divided by b (the case of $m = 0$ is the replication at baseband).

2.2. Complex sampling

Consider sampling a single-sideband (SSB) RF signal whose spectrum is shown in Fig. 2(a). Let the sampling frequency be f_s Hz, then the spectrum of the sampled signal can be obtained by replicating the spectrum of the original signal at multiples of f_s, as shown in Fig. 2(b). The complex bandpass sampled signal did not generate alias if the sampling frequency is larger than signal's bandwidth ($B = f_H$–f_L). Thus the valid sampling frequency for the SSB signal is $f_s \ge B$. Since the spectrum of the sampled signal is obtained by replicating the spectrum of the original signal at multiples of f_s, a bandpass sampled SSB RF signal which is centred at f_c will be successively replicated at the