



# Distributed estimation in diffusion networks using affine least-squares combiners <sup>☆</sup>



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## ABSTRACT

We propose a diffusion scheme for adaptive networks, where each node obtains an estimate of a common unknown parameter vector by combining a local estimate with the combined estimates received from neighboring nodes. The combination weights are adapted in order to minimize the mean-square error of the network employing a local least-squares (LS) cost function. This adaptive diffusion network with LS combiners (ADN-LS) is analyzed, deriving expressions for its network mean-square deviation that characterize the convergence and steady-state performance of the algorithm. Experiments carried out in stationary and tracking scenarios show that our proposal outperforms a state-of-art scheme for adapting the weights of diffusion networks (ACW algorithm from [10]), both during convergence and in tracking situations. Despite its good convergence behavior, our proposal may present a slightly worse steady-state performance in stationary or slowly-changing scenarios with respect to ACW due to the error inherent to the least-squares adaptation with sliding window. Therefore, to take advantage of these different behaviors, we also propose a hybrid scheme based on a convex combination of the ADN-LS and ACW algorithms.

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## 1. Introduction

Over the last years, adaptive networks have gained considerable attention as an efficient solution to estimate certain parameters of interest using the information from data collected at nodes distributed over a region (see e.g., [2–12] and their references). Many applications reach an improved behavior thanks to the use of data measured in different localizations, e.g., target localization and tracking, considering either a static or a mobile network [12,13], environment monitoring [2], and spectrum sensing in mobile networks, where secondary users can estimate the power spectrum

transmitted by all the primary users to adaptively find unused frequency bands [12,14].

In these applications, networks must track the variations in the data statistics, which justifies the need for adaptiveness [15]. It should be noted that the tracking ability of adaptive networks constitutes one of their advantages with respect to other distributed estimation schemes, such as consensus networks [12]. Additionally, some networks operate under computational, communication or energy constraints, which makes the signal processing problem even more challenging [16,17].

Different distributed processing strategies have been designed to build an estimate of some parameters of interest by exchanging information among the network nodes. In particular, in diffusion networks nodes diffuse their estimates to the network, so that each node can combine its own estimation with those received from neighboring nodes. The diffusion strategy is typically performed in two stages: adaptation and combination. The order in which these stages are performed leads to two possibilities: adapt-then-combine (ATC) and combine-then-adapt (CTA) [3,5]. In ATC, each node updates its local estimate using the combined estimate from the previous iteration. Then, the local estimates of the nodes belonging to the neighborhood are mixed to update the combined estimate. On the other hand, in CTA the combined estimate is

<sup>☆</sup> Some preliminary parts of this work appeared as a conference paper [1].

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firstly updated by mixing the estimates received from the neighboring nodes, and then used to update the local estimate. In both diffusion schemes, the combination weights play an essential role in the overall performance of the network. For instance, diffusion least-mean-squares (LMS) strategies for distributed estimation can perform similarly to classical centralized solutions when the weights used to combine the different estimates are optimally adjusted [9–12].

Most papers on diffusion networks assume fixed combination weights, whose values are computed based on the network topology only (see e.g., [3,5]). However, these static combination rules do not take into account diversity among nodes, or different signal-to-noise ratio (SNR) conditions, resulting in suboptimal performance when the SNR varies across the network. For this reason, some schemes that implement adaptive combination weights have been recently proposed [9–12,18]. Although these adaptive solutions improve the performance of the networks when compared to static combiners, the learned combination parameters may still be suboptimal during convergence or when tracking time-varying solutions, especially when different step sizes are used across the network nodes. This is due to the fact that some of the approximations used in the derivation of these adaptive rules hold mainly in steady state.

In this paper, we propose a new approach to adapt the combiners of diffusion networks which can improve network performance, especially during convergence or in tracking situations. Unlike previous schemes, the adaptive combiners that we propose here are based on the direct minimization of a least-squares cost function that considers the data available at each node. For this, we exploit some recent advances in the literature regarding combination of adaptive filters.

In an adaptive combination of filters, different schemes can be considered to mix the outputs of the component filters, including convex [19–23] and affine [24–26] combinations. In [26], a least-squares (LS) scheme was proposed to adapt an affine combination of multiple adaptive filters, providing an adequate behavior in stationary and nonstationary environments. It is important to notice that in a combination of multiple adaptive filters, all the filters (or nodes) receive the same input vector, there is a common desired signal, and the component filters do not exchange information in general,<sup>3</sup> so the extension of these combination schemes to adaptive networks is not straightforward.

As in our previous work [1], we utilize here LS-based affine combinations [26] to implement adaptive combiners in diffusion networks. This scheme, named Adaptive Diffusion Network with Least-Squares combiners (ADN-LS), presents the following characteristics:

- It follows an ATC strategy (although our results could also be straightforwardly extended to CTA). However, an important difference from standard ATC is that each node preserves a pure local estimation.
- Each node combines its local estimate with the combined estimates received from neighboring nodes at the previous iteration. Combination weights are adapted to minimize a local LS cost function employing a sliding window [26].

In this paper, we extend our workshop paper [1] in different ways. Specifically, the main contributions of this paper are:

**Table 1**

Summary of the notation used in the paper.

$N$	Number of nodes in the network
$\mathcal{N}_k$	Neighborhood of node $k$ , including itself
$N_k$	Cardinality of $\mathcal{N}_k$
$\tilde{\mathcal{N}}_k$	Neighborhood of node $k$ , excluding itself
$\tilde{N}_k$	Cardinality of $\tilde{\mathcal{N}}_k$
$\mathbf{b}_k$	Vector with the indexes of all nodes belonging to $\tilde{\mathcal{N}}_k$
$\tilde{b}_k^{(m)}$	Index of the $m$ th node connected to node $k$
$\mathbf{w}_o(n)$	Unknown time-varying parameter vector
$\hat{\boldsymbol{\psi}}_k(n)$	Local estimate of $\mathbf{w}_o(n)$ (based only on local data at node $k$ )
$\mathbf{w}_k(n)$	Combined estimate of $\mathbf{w}_o(n)$ at node $k$
$\{d_k(n), \mathbf{u}_k(n)\}$	Local desired value and regression vector at node $k$
$v_k(n)$	Local noise at node $k$ (with mean zero and variance $\sigma_v^2$ )
$c_{\ell k}(n)$	Combination weight assigned by node $k$ to the estimate of node $\ell \in \mathcal{N}_k$
$\mathbf{c}_k(n)$	Vector with all weights assigned by node $k$ to its neighbors
$\tilde{\mathbf{c}}_k(n)$	Vector with the same entries of $\mathbf{c}_k(n)$ , excluding $c_{kk}(n)$
$y_k(n)$	Local output of node $k$
$e_k(n)$	Local error of node $k$
$\hat{y}_k(n)$	Output of node $k$ using combined estimates $\mathbf{w}_k(n)$
$\tilde{e}_k(n)$	Error of node $k$ using combined estimates $\mathbf{w}_k(n)$

1. We provide a statistical analysis to model the transient and steady-state performance of ADN-LS. We arrive at analytical expressions for the mean-square deviation (MSD) and for the optimal combiners. This analysis is essential to understand the behavior of ADN-LS with optimal combiners in the transient and in the steady state, and allows us to obtain the performance limits of the proposed algorithm. To the best of our knowledge, there is no transient analysis of a diffusion scheme with adaptive combiners currently available in the literature.
2. A new hybrid scheme based on a convex combination of ADN-LS with the scheme of [10]. As we will see, ADN-LS outperforms the state-of-the-art scheme of [10], denominated as Adaptive Combination Weights (ACW) in [12], during convergence and in tracking situations. However, the steady-state performance of ADN-LS can be slightly worse than that of ACW in certain cases. In this sense, the hybrid scheme takes advantage of these somehow complementary properties.
3. A discussion on the computational and communication costs of the proposed schemes.
4. Finally, we provide detailed simulation work to illustrate the performance of our schemes. This experimental evaluation extends the results in [1] with respect to the considered algorithms and simulation scenarios. Additionally, we also study the influence of the sliding window necessary for the adaptation of ADN-LS combiners in the performance of the algorithm.

The rest of the paper is organized as follows. The next section presents the principles of adaptive diffusion networks and describes our proposal for implementing adaptive combiners that minimize an LS criterion. Then, Section 3 provides a statistical analysis of ADN-LS with optimal combiners. The hybrid scheme combining ADN-LS and ACW rules is presented in Section 4, together with an evaluation of the computational and communication costs of our schemes. Section 5 studies the stationary and tracking capabilities of our proposal, both in the illustrative case of a simple network, and for a more involved diffusion network with 15 nodes. We finish the paper by presenting our main conclusions and some possibilities for future research.

Table 1 summarizes the notation that will be used in the paper.

## 2. Diffusion networks with least-squares combiners

In this section, the ATC strategy based on the preservation of the local estimates at each node is described. Then, we present the LS adaptation for updating the adaptive combiners.

<sup>3</sup> Indeed, [19] proposed a scheme for coefficient transfer between the filters. In this scheme, the convergence of the slow filter can be accelerated when an abrupt change appears by transferring a part of the fast filter to the slow one. Therefore, a combination of adaptive filters also allows exchange of information among their components.

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