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Joint detection and estimation error bounds for an unresolved target-group using single or multiple sensors

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ABSTRACT

Joint detection and estimation (JDE) of a target refers to determining the existence of the target and estimating the state of the target, if the target exists. This paper studies the error bounds for JDE of an unresolved target-group in the presence of clutter and missed detection using the random finite set (RFS) framework. We define a meaningful distance error for JDE of the unresolved target-group by modeling the state as a Bernoulli RFS. We derive the single and multiple sensor bounds on the distance error for an unresolved target-group observation model, which is based on the concept of the continuous individual target number. Maximum a posterior (MAP) detection criteria and unbiased estimation criteria are used in deriving the bounds. Examples 1 and 2 show the variation of the bounds with the probability of detection and clutter density for single and multiple sensors. Example 3 verifies the effectiveness of the bounds by indicating the performance limitations of an unresolved target-group cardinalized probability hypothesis density (UCPHD) filter.

1. Introduction

In target tracking, most of the tracking algorithms are suboptimal because of nonlinearity, non-Gaussian noises, clutter, missed detection, and measurement origination uncertainty [1, pp. 1–52]. In order to assess the achievable performance of the suboptimal filters, we need to derive their error lower bounds, which give an indication of performance limitations. In 1998, Tichavsky et al. [2] proposed a recursive posterior Cramér–Rao lower bound (PCRLB) for evaluating the performance of existing suboptimal nonlinear filters. The PCRLB has been extended to the tracking problems where clutter is present [3–5], probability of detection is less than unity [6,7], and a target maneuvers [8].

The problem of joint detection and estimation (JDE) is an important research and has been studied in [9–11]. Tajer et al. [12] offered a new framework for joint target detection and parameter estimation using multiple-input multiple-output (MIMO) radar. Moustakides et al. [13] developed the optimum one- and two-step test for the JDE setup by combing the Bayesian formulation of the estimation sub-problem with suitable constraints on the detection sub-problem. Vo et al. [14] developed a multi-Bernoulli track before detect (TBD) filter for the JDE of non-overlapping objects using image observations with low signal-to-noise ratio (SNR).

The PCRLB rests on the assumption that the target must exist. In other words, the PCRLB only refers to the estimation error but not the detection error. Therefore, it is very difficult to be applicable to the performance evaluation of the JDE approaches. Rezaeian et al. [15] derived the single sensor mean square error (MSE) bounds for the JDE of a single point target based on the random finite set (RFS). Tong et al. [16] presented recursive forms of error bounds for the RFS state and observation when the probability of detection is less than unity without clutter. The research on error bounds is very valuable for many real-world applications, such as sensor management [17], ballistic target tracking [18], performance prediction, and passive tracking [19,20].

Unresolved targets (or target-groups) tracking (UTT), proposed first by Drummond, Blackman and Petrisor in 1990 [21], has become an important research in recent years. The problem of UTT arises because of the sensor resolution, high normalized target density, and sensor-to-target geometry [22,23]. An unresolved target-group refer to a cluster of two or more closely spaced individual targets, which cannot be completely resolved due to finite sensor resolution. In other words, treating a cluster of indistinguishable point targets as an entire object (especially if they move in a coordinated fashion according to a certain constraint or interrelationship, it is usually reasonable to treat them as an entire

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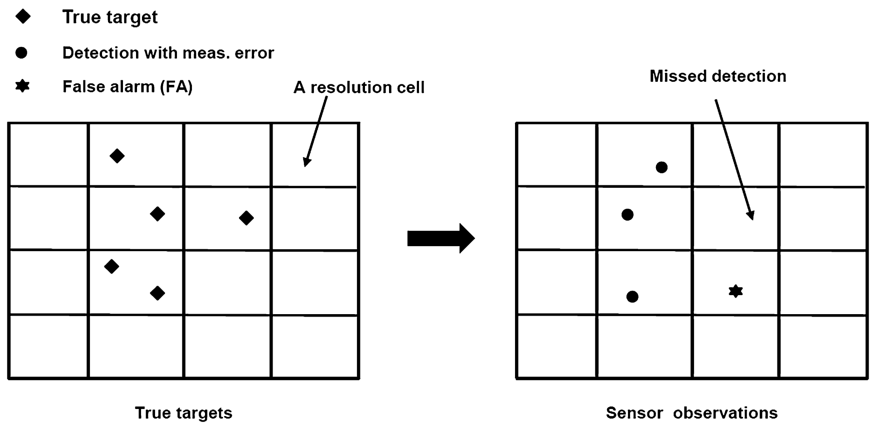

Fig. 1. A sensor observes five point targets. Two targets lie in the same resolution cell and are not resolved. The sensor produces detections with measurement errors. One target is not detected due to missed detection and a false alarm appears due to clutter. We can say that the five point targets form an unresolved target-group.

object), they are idealized as an unresolved target-group [24]. In [25, pp. 432–433], an unresolved target-group is also called a "point target cluster". In general, the number of detections generated by target-group will be different from the number of individual targets in the target-group due to finite sensor resolution, sensor missed detection, and clutter as shown in Fig. 1.

Above all, an unresolved target-group of this paper is an entire object rather than only one point target. It is formed by a cluster of indistinguishable point targets.

Many approaches have been proposed for UTT. A comprehensive overview of existing approaches to UTT up to 2004 is provided in [26]. Blair and Keel discuss the significance of finite radar resolution in the context of UTT in [23]. Koch et al. [27,28] devised an approximate Bayesian solution to jointly estimating the geometric centroid and ellipsoidal shape of an unresolved target-group or extended target. A new variant of probabilistic multiple hypotheses tracker (PMHT) [29], which permits that multiple measurements originate from the same object, could be applied for UTT. Blom et al. [30] and Jeong et al. [31], respectively, studied the single and multiple sensor Bayesian filtering algorithms for maneuvering UTT. Nandakumaran et al. [32] and Gorji et al. [33], respectively, proposed JDE algorithms for UTT using monopulse radar and colocated MIMO radar. Within the RFS framework, Mahler developed a novel probability hypothesis density (PHD) filter for JDE of unresolved target-groups [34]. Then, the authors extended the method to unresolved target-group cardinalized PHD (UCPHD) filter [35] and unresolved target-group multi-Bernoulli filter [36]. Mihaylova et al. [37] presented an overview of Bayesian sequential Monte Carlo methods for UTT. However, to the best of our knowledge, there has been no study on the JDE error bounds for UTT until now.

In this paper, we first present a meaningful distance error for the JDE of an unresolved target-group by modeling the state as a Bernoulli RFS. An observation model based on the concept of continuous target number is used to describe the likelihood function for the unresolved target-group according to [25, pp. 437–440]. Then we derive the single and multiple sensor bounds on the distance error for JDE of an unresolved target-group in the presence of missed detection and clutter. Maximum a posterior (MAP) detection criteria and unbiased estimation criteria are used in deriving the bounds. Finally, three numerical examples are presented using simulated data. Examples 1 and 2 show the variation of the bounds with probability of detection and clutter density in the cases of single sensor and multiple sensors. Example 3 verifies the effectiveness of the bounds by indicating the performance limitations of a UCPHD filter [35].

In the current set up of this paper, our attention is restricted to the static JDE problem of a single unresolved target-group. Our future work will study the recursive extensions to the filtering problems by consideration of the unresolved target-group state evolution.

The rest of the paper is organized as follows. Section 2 describes the problem for JDE of an unresolved target-group based on the RFS. In Sections 3 and 4, we present the single and multiple sensor MSE bounds for the JDE of an unresolved target-group, respectively. We present three numerical examples in Section 5. Conclusions and future work are given in Section 6. Relevant mathematical proofs of these conclusions are presented in Appendices A–D.

2. Problem statement for JDE of an unresolved target-group based on RFS

An unresolved target-group state is modeled by an augmented vector of form

$$\boldsymbol{x}' = (a, \boldsymbol{x}) \in \mathcal{X}', \tag{1}$$

where the l-dimensional vector $\boldsymbol{x} = [x_1, \ldots, x_l]^T \in \mathcal{R}^l$ denotes a conventional single-target state, a denotes the expected number of individual targets involved in the unresolved target-group. Here we assume that the dimension of the state for each individual target in the group is the same. $\boldsymbol{x}'$ models a cluster of a individual targets colocated at the single state $\boldsymbol{x}$. According to the concept of continuous individual target number proposed by Mahler [25, pp. 437–440], $a \in \mathcal{A} \subset \mathcal{R}^+$ is a positive real number. $\mathcal{X}'$ denotes the state space of a single unresolved target-group, $\mathcal{X}' \subset \mathcal{R}^+ \times \mathcal{R}^l$.

We consider the problem of JDE of an unresolved target-group within the Bayesian framework. Therefore, the prior knowledge about the unresolved target-group must be used. We assume that the number of the unresolved target-groups in the surveillance area is at most one. Therefore, it is suitable to model the state set X' of the unresolved target-group as a Bernoulli RFS $X' \sim \mathcal{B}_{X'}(r, f(\boldsymbol{x}'))$ with the probability density

$$\pi(X') = \begin{cases} 1 - r, & \text{for } X' = \emptyset, \\ r f(\boldsymbol{x}'), & \text{for } X' = \{\boldsymbol{x}'\}, \end{cases} \tag{2}$$

where $r \in [0, 1]$ denotes the probability of $X' \neq \emptyset$, $f(\boldsymbol{x}')$ (defined on $\mathcal{X}'$) denotes the probability density of $\boldsymbol{x}'$. r represents the prior existence probability of an unresolved target-group and $f(\boldsymbol{x}')$ represents the prior knowledge about state $\boldsymbol{x}'$ if it exists. Therefore, the Bernoulli density $\pi(X')$ encapsulates all the prior information about the unresolved target-group.