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Effects of underdamped step-varying second-order stochastic resonance for weak signal detection

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A B S T R A C T

Stochastic resonance (SR) has been proved to be an effective approach for weak signal detection. In this paper, an underdamped step-varying second-order SR (USSSR) method is proposed to further improve the output signal-to-noise ratio (SNR). In the method, by selecting a proper underdamped damping factor and a proper calculation step, the weak periodic signal, the noise and the potential can be matched with each other in the regime of second-order SR to generate an optimal dynamical system. The proposed method has three distinct merits as: 1) secondary filtering effect produces a low-noise output waveform; 2) good band-pass filtering effect attenuates the multiscale noise that locates in high- and (or) low-frequency domains; and 3) good anti-noise capability in detecting weak signal being submerged in heavy background noise. Numerical analysis and application verification are performed to confirm the effectiveness and efficiency of the proposed method in comparison with a traditional SR method.

1. Introduction

Since proposed by Benzi et al. in 1981, stochastic resonance (SR) has become a hot research topic in the field of nonlinear science [1]. The most distinct merit of SR is that the weak continuous signal can be enhanced by exploiting the noise energy [2]. SR has been widely adopted in amplification of weak signals in different research fields. For instance, SR and optimal detection of pulse trains by threshold devices was introduced in Ref. [3]. Signal amplification in a nanomechanical Duffing resonator via SR was proposed in Ref. [4]. Signal amplification and filtering with a tristable SR cantilever was suggested in Ref. [5]. Nonlinear signal detection via SR was presented in Refs. [6,7]. Comparison study between the SR and the matched filters in detecting bipolar pulse signals was reported in Ref. [8]. Condition for noise induced enhancement in weak signal detection via SR was studied in Ref. [9]. Detection performance via SR in hypothesis-testing problems in the Neyman–Pearson framework was discussed in Refs. [10,11]. These studies indicate that SR phenomenon exists in different signal systems which contain weak signals and noise, and the weak signals can be enhanced and then detected by the assistance of proper noise.

The majority of SR theories are developed in the framework of small parameter (both signal frequency and signal amplitude should be far less than one) [12]. However, in practical signal processing applications, e.g., machine fault diagnosis, the small parameter limitation of the classical SR is hardly satisfied, and additionally, the engineering signals always present the characteristics of nonlinearity and nonstationarity. Hence, to make the classical SR suitable for addressing the large parameter signals, a lot of modified and optimized strategies have been proposed, such as re-scaling frequency SR [13], adaptive step-changed SR [14], frequency-shifted and re-scaling SR [15], multiscale noise tuning SR [16] and multi-scale bistable SR array [17], etc. These studies are realized by tuning the signal structure and (or) the system parameters to cope with the large parameter signal, and these efforts have effectively promoted the SR-based weak signal detection techniques in the applications of practical engineering signal processing.

However, most of engineering signal processing methods via SR principle are based on the simplest first-order overdamped SR models, i.e., the system inertia is ignored and the system damping effect is regarded as insignificant (and hence the damping factor is normalized for simplicity). In fact, the SR output signal can be seen as a particle trajectory induced by particle oscillation within a potential under the synthetic forces from the weak signal and the noise [18]. From this perspective, the system inertia and the system damping factor will have effects on SR realization. Considering the system inertia means that the SR model is second-order, and actually, the second-order SR is beneficial to obtain a low-noise SR output signal as the SR procedure can be regarded as a specific signal filtering process (i.e., second-order SR means

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secondary filtering and hence produces a cleaner filtered signal than first-order SR) [19]. Furthermore, several studies indicate that the underdamped damping factor also affects the efficiency of particle oscillation in the potential and finally affects the SR performance [20–25]. Thus, considering a second-order SR model with an underdamped damping factor provides a possibility in improving the SR-based weak signal detection effect.

Motivated by the aforementioned analysis, this study focuses on enhancement of weak signal detection by combining a system parameter tuning method (via changing calculation step [14]) and an underdamped second-order SR model. This proposed method, called underdamped step-varying second-order SR (USSSR), is realized by selecting a proper calculation step and a proper underdamped damping factor in the framework of second-order SR. These two optimal parameters make the system, the driving signal and the noise be matched with each other, and thus the weak signal can be extracted from the background noise to an extreme and finally the optimal output signal can be obtained. The USSSR method has three distinct merits as: 1) secondary filtering effect produces a low-noise output waveform; 2) good band-pass filtering effect attenuates the multiscale noise that locates in high- and (or) low-frequency domains; and 3) good anti-noise capability in detecting weak signal being submerged in heavy background noise. Hence, the proposed method is expected to be extensively used in weak signal detection, especially for signals with large parameters and (or) being subjected to multiscale heavy noise interference.

The rest of this paper is organized as follows. Section 2 provides the theoretical background of the proposed USSSR method and introduces the weak signal detection strategy based on the USSSR method. Section 3 performs simulation analysis to evaluate the USSSR method in comparison with a traditional SR method. Section 4 verifies the practicability of the proposed method by analyzing a set of defective bearing signals and provides further discussions. Section 5 summarizes this paper.

2. USSSR

2.1. Basic model

The basis of classical bistable SR phenomenon can be described as: a particle is driven by a periodic signal and the random noise in a bistable potential which consists of two potential wells and one potential barrier, and the periodic oscillation can be enhanced by the assistance of proper noise. Such a phenomenon can be illustrated with a governing equation by considering both the system inertia and the system damping factor as below:

$$\frac{d^2x}{dt^2} = -\frac{dU(x)}{dx} - \gamma\frac{dx}{dt} + S(t) + N(t) \tag{1}$$

where $N(t) = \sqrt{2D}\xi(t)$ with $\langle N(t), N(t+\tau)\rangle = 2D\delta(t)$ being the noise item, in which D is the noise intensity and $\xi(t)$ represents an additive Gaussian white noise (AGWN) with zero mean and unit variance. $S(t) = A\cos(\Omega t + \phi)$ is a periodic signal, in which A is the amplitude, $\Omega = 2\pi f_d$ with f_d being the driving frequency, and ϕ is the phase. γ represents the system damping factor. $U(x)$ is a reflection-symmetric quartic potential as written below:

$$U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4 \tag{2}$$

in which a and b denote the barrier parameters of the bistable potential with positive real value. Substitute Eq. (2) into Eq. (1), then we can get the following equation:

$$\frac{d^2x}{dt^2} = ax - bx^3 - \gamma\frac{dx}{dt} + A\cos(2\pi f_d t + \varphi) + \sqrt{2D}\xi(t) \tag{3}$$

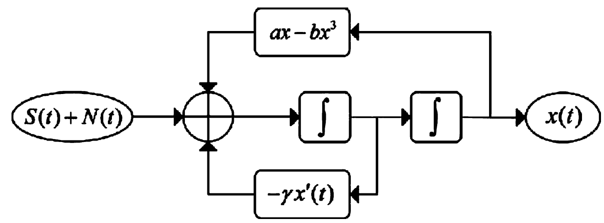

Fig. 1. The system model of the underdamped second-order SR.

The system model corresponding to Eq. (3) can be illustrated in Fig. 1, where it can be seen that the calculation of SR output $x(t)$ is equivalent to a secondary integration process and also equivalent to a secondary filtering process.

2.2. SNR analysis

Subsequently, the effect of second-order SR for enhancing the weak signal by exploiting the noise energy within the regime of small parameter is discussed. For simplicity, we set $\gamma = 0$, $\phi = 0$ and mathematically let $\frac{dx}{dt} = y$, then Eq. (3) can be separated into two first-order differential equations as:

$$\begin{aligned}\frac{dx}{dt} &= y\\ \frac{dy}{dt} &= ax - bx^3 - \gamma y + A\cos(\Omega t) + \sqrt{2D}\xi(t)\end{aligned} \tag{4}$$

Next, let $A = 0$, $D = 0$, $\frac{dx}{dt} = 0$, $\frac{dy}{dt} = 0$, then three singular points $((x_+, y_+) = (\sqrt{(a/b)}, 0), (x_0, y_0) = (0, 0), (x_-, y_-) = (-\sqrt{(a/b)}, 0))$ of the bistable potential can be obtained. Linearize Eq. (4) at singular points (x_+, y_+) and (x_-, y_-) then obtain the linearization matrix $\left[\begin{smallmatrix}0 & 1\\ -2a & 0\end{smallmatrix}\right]$, and the corresponding eigenvalues can be calculated to be $\beta_{1,2} = \pm\sqrt{-2a}$. Analogously, linearize Eq. (4) at singular point (x_0, y_0) then obtain the linearization matrix $\left[\begin{smallmatrix}0 & 1\\ a & 0\end{smallmatrix}\right]$ and the corresponding eigenvalues $\lambda_{1,2} = \pm\sqrt{a}$ [19]. Obviously, the singular point (x_0, y_0) is an unstable saddle point as $\lambda_1 > 0$ and $\lambda_2 < 0$, and the stable and unstable manifolds at (x_0, y_0) can be formed by the stable and unstable orbits that cross the saddle point, respectively. The stable manifold forms two boundaries of the stable attraction domains and the unstable manifold connects the three singular points. Subsequently, the probability density function $\rho(x, y, t)$ of the particle motion can be deduced by consulting the Fokker–Planck equation as follows [26]:

$$\begin{aligned}\frac{\partial}{\partial t}\rho(x, y, t) = &-\frac{\partial}{\partial x}\big[y\rho(x, y, t)\big]\\ &-\frac{\partial}{\partial y}\big[\big(ax - bx^3 + A\cos\Omega t\big)\rho(x, y, t)\big]\\ &+D\left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}\right)\rho(x, y, t)\end{aligned} \tag{5}$$

In addition, the quasi-steady-state distribution function $\rho_{st}(x, y, t)$ corresponding to Eq. (5) can be further obtained based on the adiabatic elimination theory as [12,27]:

$$\rho_{st}(x, y, t) = \bar{N}\exp\left[-\frac{\tilde{U}(x, y, t)}{D}\right] \tag{6}$$

in which $\bar{N}$ represents the normalization constant, and $\tilde{U}(x, y, t)$ is the generalized potential function that can be obtained by utilizing the small parameter expansions method as:

$$\tilde{U}(x, y, t) = \frac{1}{2}y^2 - \frac{a}{2}x^2 + \frac{b}{4}x^4 - xA\cos(\Omega t) \tag{7}$$