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Optimization of signal-to-noise-plus-distortion ratio for dynamic-range-limited nonlinearities

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ABSTRACT

Many components used in signal processing and communication applications, such as power amplifiers and analog-to-digital converters, are nonlinear and have a finite dynamic-range. The nonlinearity associated with these devices distorts the input, which can degrade the overall system performance. Signal-to-noise-plus-distortion ratio (SNDR) is a common metric to quantify the performance degradation. One way to mitigate nonlinear distortions is by maximizing the SNDR. In this paper, we analyze how to maximize the SNDR of the nonlinearities in optical wireless communication (OWC) systems. Specifically, we answer the question of how to optimally predistort a double-sided memory-less nonlinearity that has both a "turn-on" value and a maximum "saturation" value. We show that the SNDR-maximizing response given the constraints is a double-sided limiter with a certain linear gain and a certain bias value. Both the gain and the bias are functions of the probability density function (PDF) of the input signal and the noise power. We also find a lower bound of the nonlinear system capacity, which is given by the SNDR and an upper bound determined by dynamic signal-to-noise ratio (DSNR). An application of the results herein is to design predistortion linearization of nonlinear devices like light emitting diodes (LEDs).

1. Introduction

In addition to being nonlinear, many components in a signal processing or communication system have a dynamic-range constraint. For example, light emitting diodes (LEDs) are dynamic-range-constrained devices that appear in intensity modulation (IM) and direct detection (DD) based optical wireless communication (OWC) systems [1,2]. To drive an LED, the input electric signal must be positive and exceed the turn-on voltage of the device. On the other hand, the signal is also limited by the saturation point or maximum permissible value of the LED. Thus, the dynamic-range constraint can be modeled as two-sided clipping. The same situation may happen in other applications such as digital audio processing [3].

Both nonlinearity and clipping result in distortions which may cause system performance degradation. Signal-to-noise-plus-distortion ratio (SNDR) is a commonly used metric to quantify the distortion that is uncorrelated with the signal [4–7]. Previous work in this area mainly concentrated on a family of amplitude-limited nonlinearities that is common in radio frequency (RF) system design involving nonlinear components such as power amplifiers (PAs) and mixers.

Different from the previous work, our study discusses the class of nonlinearities with a two-sided dynamic-range constraint that is more commonly found in optical and acoustic systems. The authors in [8–12] illustrated the impact of LED nonlinearity and clipping noise in OWC systems. Some predistortion strategies were proposed in [13–15]. However, to the best of our knowledge, the optimal nonlinear mapping under the two-sided dynamic-range constraint has not been studied.

We consider two major differences from amplitude-limited nonlinearity. First, the signal will be subject to turn-on clipping and saturation clipping to meet the dynamic-range constraint. Second, dc biasing must be used to shift the signal to an appropriate level to minimize distortion. In this paper, we will show that the ideal linearizer that maximizes the SNDR is a double-sided limiter that has an affine response. The parameters of the response can be calculated from the distribution of the input signal and the noise power.

In addition to deriving the SNDR-optimal predistorter, we also relate a lower bound on channel capacity to the SNDR, further motivating the SNDR considerations. Finally, we employ another common distortion metric, dynamic signal-to-noise ratio (DSNR) to provide an upper bound on the double-sided clipping channel.

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The remainder of this paper is organized as follows: Section 2 introduces the system model for dynamic-range-limited nonlinearity and the corresponding SNDR definition. In Section 3, we derive the optimal nonlinear mapping that maximizes the SNDR and illustrate some examples. In Section 4, we relate the SNDR to the capacity of the nonlinear channel. Finally, Section 5 concludes the paper. Detailed proofs in this paper are deferred to the Appendices.

2. System model and SNDR definition

2.1. System model

Let us consider a system modeled by

$$y_o(t) = h_o\big[x_o(t)\big] + v(t) \tag{1}$$

where $x_o(t)$ is a real-valued signal with mean μ_x and variance σ_x^2; $v(t)$ is a zero-mean additive noise process with variance σ_v^2; $h_o(\cdot)$ is a memoryless nonlinear mapping with dynamic-range constraint $A_1 \le h_o[x_o(t)] \le A_2$.

For notational simplicity, we omit the t-dependence in the memoryless system and replace $h_o(\cdot)$ and $x_o(t)$ by $h(\cdot) = h_o(\cdot) - A_1$ and $x = x_o - \mu_x$. Then we have an equivalent system modeled by

$$y = h(x) + v \tag{2}$$

where $h(\cdot)$ is a memoryless nonlinear mapping with dynamic-range constraint $0 \le h(x) \le A = A_2 - A_1$ and x is a zero-mean signal with variance σ_x^2.

2.2. SNDR definition

According to Bussgang's Theorem [16], the nonlinear mapping in (2) can be decomposed as

$$h(x) = \alpha x + d \tag{3}$$

where d is the distortion caused by $h(\cdot)$ and α is a constant, selected so that d is uncorrelated with x, i.e., $E[xd] = 0$. Thus

$$\alpha = \frac{E[xh(x)] - E[xd]}{E[x^2]} = \frac{E[xh(x)]}{E[x^2]} = \frac{E[xh(x)]}{\sigma_x^2}. \tag{4}$$

The distortion power is given by

$$\begin{aligned} \varepsilon_d &= E\big[d^2\big] - E^2[d] \\ &= E\big[h^2(x)\big] - \alpha^2\sigma_x^2 - E^2\big[h(x)\big], \end{aligned} \tag{5}$$

where we use the notation $E^2(\cdot) = [E(\cdot)]^2$. The signal-to-noise-plus-distortion ratio (SNDR) is defined as

$$\begin{aligned} \mathrm{SNDR} &= \frac{\alpha^2\sigma_x^2}{\varepsilon_d + \sigma_v^2} \\ &= \frac{E^2[xh(x)]/\sigma_x^2}{E[h^2(x)] - E^2[xh(x)]/\sigma_x^2 - E^2[h(x)] + \sigma_v^2}. \end{aligned} \tag{6}$$

The definition of SNDR here is a little bit different from that in [7], because all the signals are real and the distortion contains dc biasing. Thus, the distortion power is modeled as variance rather than the secondary moment.

We see from (6) that the SNDR is related to the distribution of x, the noise power σ_v^2, and the nonlinear mapping $h(\cdot)$. Our aim in the next section is to determine the function $h(\cdot)$ that maximizes the SNDR given a signal distribution and the two-sided clipping constraint.

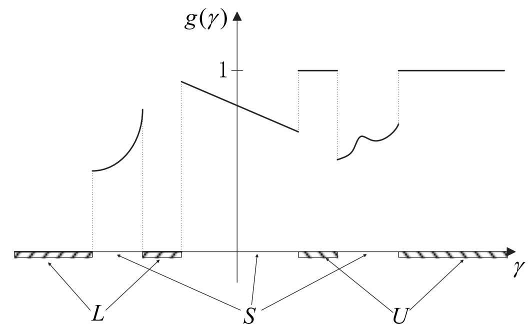

Fig. 1. An example of nonlinear mapping $g(\cdot)$ that satisfies the $0 \le g(\cdot) \le 1$ constraint.

3. SNDR optimization and examples

3.1. Optimization of SNDR

Similar to [7], let us use a function $g(\cdot)$ to normalize the nonlinear mapping $h(\cdot)$:

$$h(x) = Ag\left(\frac{x}{\sigma_x}\right) \tag{7}$$

where $0 \le g(\cdot) \le 1$. Let $\gamma = x/\sigma_x$ and substitute (7) into (6) to obtain

$$\begin{aligned} \mathrm{SNDR} &= \frac{E^2[\gamma g(\gamma)]}{E[g^2(\gamma)] - E^2[\gamma g(\gamma)] - E^2[g(\gamma)] + \sigma_v^2/A^2} \\ &= \frac{E^2[\gamma g(\gamma)]}{\mathrm{var}[g(\gamma)] - E^2[\gamma g(\gamma)] + \sigma_v^2/A^2} \end{aligned} \tag{8}$$

where $\mathrm{var}[g(\gamma)] = E[g^2(\gamma)] - E^2[g(\gamma)]$ is the variance of $g(\gamma)$.

The SNDR optimization problem can be stated as follows:

$$\max_{\substack{g(\cdot) \\ 0 \le g(\cdot) \le 1}} \quad \mathrm{SNDR} \tag{9}$$

for a given distribution of γ, dynamic-range A and noise power σ_v^2.

Fig. 1 illustrates an example of the $g(\cdot)$, where the region of γ is divided into three sets L, S and U:

$$g(\gamma) = 0, \quad \text{for } \gamma \in L; \tag{10}$$

$$0 < g(\gamma) < 1, \quad \text{for } \gamma \in S; \tag{11}$$

$$g(\gamma) = 1, \quad \text{for } \gamma \in U. \tag{12}$$

Thus, to determine a nonlinear mapping $g(\cdot)$, we need to find the sets L, S, U and the shape of the function $g(\cdot)$ in S.

We will solve this problem with the following steps:

1. find the optimal $g(\cdot)$ given L, S, U;
2. show that S should be as large as possible;
3. determine L and U for the optimal solution.

Lemma 1. *Assume that the sets L, S and U are known, and $L \cup S \cup U = R$. The $g(\cdot)$ function that maximizes the SNDR expression in (8) is of the form*

$$g(\gamma) = \frac{\gamma}{\eta} + \beta \tag{13}$$

where