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Modeling and estimation of single-trial event-related potentials using partially observed diffusion processes

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ABSTRACT

This paper proposes a new modeling framework for estimating single-trial event-related potentials (ERPs). Existing studies based on state-space approach use discrete-time random-walk models. We propose to use continuous-time partially observed diffusion process which is more natural and appropriate to describe the continuous dynamics underlying ERPs, discretely observed in noise as single-trials. Moreover, the flexibility of the continuous-time model being specified and analyzed independently of observation intervals, enables a more efficient handling of irregularly or variably sampled ERPs than its discrete-time counterpart which is fixed to a particular interval. We consider the Ornstein–Uhlenbeck (OU) process for the inter-trial parameter dynamics and further propose a nonlinear process of Cox, Ingersoll & Ross (CIR) with a heavy-tailed density to better capture the abrupt changes. We also incorporate a single-trial trend component using the mean-reversion variant, and a stochastic volatility noise process. The proposed method is applied to analysis of auditory brainstem responses (ABRs). Simulation shows that both diffusions give satisfactory tracking performance, particularly of the abrupt ERP parameter variations by the CIR process. Evaluation on real ABR data across different subjects, stimulus intensities and hearing conditions demonstrates the superiority of our method in extracting the latent single-trial dynamics with significantly improved SNR, and in detecting the wave V which is critical for diagnosis of hearing loss. Estimation results on data with variable sampling frequencies and missing single-trials show that the continuous-time diffusion model can capture more accurately the inter-trial dynamics between varying observation intervals, compared to the discrete-time model.

1. Introduction

Event-related potentials (ERPs) are bioelectrical responses of the brain to specific stimulus, which provide useful information about neurological processing. The problem is to extract the ERPs hidden behind various noise sources e.g. background electroencephalogram (EEG) and non-neural artifacts, typically in poor signal-to-noise ratio (SNR) condition. Conventional ensemble averaging of time-locked single-trials obtained by repeated stimulations can cancel out random background noise, but implies loss of information related to trial-to-trial variability due to different degree of fatigue, habituation, or attention levels of subjects [1]. Auditory brainstem response (or evoked potentials) (ABRs or ABEPs), a particular type of ERPs is the early portion of the auditory evoked potentials. The major waves of ABR are important for hearing and neurological assessment and exhibit variability across trials. Estimation of inter-trial variability and common trend of major waves for these small-amplitude potentials obscured by various noise and artifacts, poses a challenge to this study.

Various approaches were proposed to solve single-trial based estimation of ERPs which is also the focus of this study. State-space approach was recently proposed for single-trial dynamical ERP estimation in [1–4] and our work [5]. Within the state-space framework, the underlying dynamics of clean ERPs are modeled as a discrete-time first-order Markov process i.e. a simple Gaussian random-walk process in the state equation, which is observed in additive Gaussian noise as single-trial measurements in the observation equation. The latent ERP process is then estimated recursively using Kalman filter (KF) [1], which is optimal in the mean-square sense. Extension using Kalman smoother [2] further improves the performance in tracking of ERP changes. Similar study [3] proposed a particle filter (PF) for the estimation and uses wavelet coefficients, instead of measurement samples, as ERP parameters [1] to reduce the state dimension and hence

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the computational effort. This study was extended in [4], which models the ERP subcomponent waveforms using shifted Gaussian functions, which can directly characterize important ERP morphological features such as the peak amplitudes and latencies. The Gaussians are also less inclined to capture the abruptly changing random noises, except the usually smooth ERP waves. The parameter estimation is done by Rao–Blackwellised PF (RBPF). However, the above-mentioned studies [1–5] assume discrete-time modeling which is inappropriate to describe the single-trial ERPs, the underlying physiological processes which are typically of continuous nature. Besides, the discrete-time models are unable to handle the irregularly sampled data problem which is common in practice, where the ERP measurements are unequally spaced with some single-trials occasionally missing, probably due to sensor failures. Moreover, these studies use a Gaussian noise model with constant variance, which fails to describe adequately the actual observational noise process whose volatility typically changes over time.

We develop a new framework for modeling the inter-trial dynamics of ERPs based on continuous-time partially-observed (PO) diffusion process, with applications to estimation of single-trials corrupted by noise. Our approach is rooted in the state-space framework of the aforementioned studies, but offers several enhancements over the existing models, by relaxing the inadequate discrete-time and constant noise-volatility modeling assumptions and incorporating additional, important features of ERPs which have not so far been accounted for, such as the trend of the single-trials. In particular, we decompose the single-trial observations into three components: (1) latent clean ERP component, (2) observational noise component and (3) trend component, each described by a separate stochastic process. For the first component, we propose to use continuous-time processes to model the underlying dynamics of the latent clean ERPs. The continuous-time models have been applied extensively in physics, biology, econometrics [6,7] and engineering [8], due to their advantages over their discrete-time counterparts. This also forms our motivation of applying it for modeling single-trial ERPs, with the following reasons. First, it seems more natural and appropriate to use continuous-time modeling to describe real-world dynamical phenomena [7], such as the continuous physiological process underlying the single-trial EPRs considered here. Despite that the observations are always available only at discrete-time points; it is more adequate to assume that they are generated from continuous processes. Secondly, the continuous-time models can deal with variably or irregularly sampled data problem efficiently [8,9], due to their flexibility in defining implicitly and consistently over observation intervals of any length, with their adjustable parameter space estimated independently of the data sampling interval. However, the discrete-time models are specified in relation to a particular sampling interval, and therefore are fixed once estimated and only valid for this chosen sampling rate. Thus, the variable sampled data needs to be incorporated explicitly through model re-estimation. We illustrate these advantages of the continuous-time approach over its discrete-time counterpart, on two real situations in ERP estimation: (1) Estimation with variably sampled (down-sampled) single-trials, which can save computational effort without affecting the overall estimated dynamic pattern of ERPs; and (2) Estimation with irregularly spaced single-trials with missing observations where some trials contains very low (sometimes zeros) signals, which are currently solved only by direct elimination [10].

We consider the commonly used continuous-time model i.e. the diffusion process driven by Brownian noise. Diffusion processes governed by the stochastic differential equations (SDEs) can accommodate random disturbances in the deterministic behavior, and are widely used for modeling physical and biological dynamical systems disturbed by noise [11]. Besides, the diffusion process is a continuous-time Markovian process to be contrasted with the discrete-time random-walk process used previously in [1–5]. Various variants of diffusion models with rich modeling properties have been proposed. To accommodate the discrete-time observation scheme of single-trial ERPs, we use a particular family of PO diffusion processes where the continuous-time diffusions are discretely observed [11], possibly further with noise [12]. Such formulation is suitable to describe the continuous dynamics underlying ERPs which are observed only as discrete-time noisy single-trials. Specifically, the hidden continuous dynamic changes in the ERP Gaussian mixture parameters are assumed to follow a multivariate diffusion process, which is only partially observed at discrete times with additive background noise as single-trial data in the observation model. To the authors' knowledge, there are no studies applying this kind of models for analyzing ERPs and its use for bio-signals in general is still very limited.

A simple special case of diffusion process is first considered, i.e. the Ornstein–Uhlenbeck (OU) process. The use of OU model in its linear Gaussian form, assumes marginal inter-trial ERP changes that can be captured by its normal transition density. However, this assumption is inconsistent with the presence of both smooth and abrupt changes in the actual ERP dynamics, which implies a non-Gaussian behavior. The OU model is unable to capture both of these changes simultaneously, where a small variance of the normal transition density fails to detect rapidly the abrupt changes, while large variance tends to produce noisy estimates. This Gaussian modeling problem has been addressed in [13] and our earlier work [14] for other types of models in discrete-time. Another limitation of the OU model is that it allows for a negative process which is unsuitable for the non-negative latency and width parameters of the Gaussian ERP components. To accommodate these non-Gaussian dynamics of ERPs, we further propose an enhancement over the linear Gaussian OU process by considering a non-linear diffusion process of Cox, Ingersoll & Ross (CIR) [15]. The non-Gaussian heavy-tailed transition density of CIR process, a non-central chi-squared density can capture both abrupt and smooth ERP parameter changes, hence providing more accurate single-trial estimates than the OU model. We use the Gaussian OU process which can take negative values to model the amplitude parameters, and the non-negative CIR process for the latency and width parameters. Moreover, both diffusions are stationary and thus produce more stable ERP estimates than the non-stationary random-walk models of [1–4].

To incorporate the single-trial trend component, we proceed to use the mean-reverting variant for both diffusion processes, which approaches a stationary asymptotic distribution in long run. The advantage is that the asymptotic mean of the stationary distribution can represent the underlying trend of the trial-varying ERPs, revealing more clearly the overall morphology of the ERP waveform. To capture more accurately the changing-volatility in the noise component in order to achieve better noise-reduction, we allow the observational noise variance to be varying over trials instead of a constant one. We consider the stochastic volatility (SV) modeling [16] where the trial-varying variance is modeled directly as a latent stochastic process, which is flexible and easy to generalize to multivariate case. The idea of incorporating SV noise models for denoising of single-trial ERPs has been explored in our recent work [17]. Preliminary evaluation on normal ERPs of a single-subject showed more accurate estimates of inter-trial dynamics compared to the constant noise variance. The evolution of both the trend and noise components is assumed to follow a simple random-walk process.

The proposed PO diffusion model of ERPs is then reformulated and estimated under the state-space framework. Estimation of the non-linear processes of latency and width parameters admits no closed-form solution and has to resort to simulation-based