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Delay-dependent $\mathscr{H}_\infty$ filtering of a class of switched discrete-time state delay systems

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ABSTRACT

In this paper, the problem of delay-dependent $\mathscr{H}_\infty$ filtering for a class of discrete-time switched systems with state delays is investigated under arbitrary switching using a parametrized approach. Attention is focused on the design of asymptotically stable filter guaranteeing a prescribed noise attenuation level in the $\mathscr{H}_\infty$ sense. By using switched Lyapunov functionals, sufficient conditions for the solvability of this problem are obtained in terms of linear matrix inequalities (LMIs), the solution of which constructs the desired $\mathscr{H}_\infty$ filter under arbitrary switching. Numerical examples are provided to demonstrate the effectiveness of the proposed techniques.

1. Introduction

It is well known that state estimation has been widely studied and has found many practical applications during the past decades. When a priori information on the external noise is not precisely known, the celebrated Kalman filtering scheme is no longer applicable. In this case, $\mathscr{H}_\infty$ filter was introduced where the noise signal was assumed to be energy bounded and the main objective was to minimize the H_∞ norm of the filtering error system [1–5,36]. When time delays are taken into account in a system, linear matrix inequality (LMI)-based results on the H_∞ filtering problem have also been reported in the literature; see, e.g., [1,6–8,35] and the references therein.

Recently, the control synthesis of switched systems has been extensively investigated and many methodologies have been used in the study of switched systems [9–17]. For example, multiple Lyapunov functions were employed to establish certain general Lyapunov-like results for nonlinear switched systems [11], dwell-time and average dwell-time approaches were employed to study the stability and disturbance attenuation of switched systems [18,19], piecewise Lyapunov function approach was adopted in [20], and a switched Lyapunov function method has been applied in [9] to study the stability problem of discrete time switched systems.

On the other hand, time delays are the inherent features of many physical process and the big sources of instability and poor performances [21,22,37]. Switched systems with time delays have strong engineering background in network control systems [13] and power systems [23]. More recently, some theoretical studies were conducted for switched systems with time delays [24,25]. A delay-independent approach to $\mathscr{H}_\infty$ filtering problem for time-delayed switched systems has been addressed in [26,27].

In this paper, a delay-dependent $\mathscr{H}_\infty$ filtering design is established by using switched Lyapunov functional approach for a class of discrete-time switched systems with bounded time-varying delays. The objective of our work is to develop a

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less-conservative conditions to guarantee the closed-loop system stability under arbitrary switching. The filtering design solution is facilitated by introducing some additional instrumental matrix variables. These additional matrix variables decouple the Lyapunov and the system matrices, which makes the filtering design feasible.

The organization of this paper is as follows. Section 2 formulates the $\mathscr{H}_\infty$ filtering problem of the switched system. The analysis of stability and $\mathscr{H}_\infty$ performance for the filtering error system is given in Section 3. The filtering design is given in Section 4. A numerical example is given to illustrate the effectiveness of the results in Section 5, and then Section 6 concludes the paper.

Notations: Throughout this paper, a real symmetric matrix $P>0$ denotes P being a positive definite matrix, and $A>B$ means $A-B>0$, M^{T} denotes the transpose of the matrix M. I is used to denote an identity matrix with appropriate dimension. The notation $l_2[0,\infty)$ refers to the space of square summable infinite vector sequences with the usual norm $\|\cdot\|_2$. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem formulation

Consider the following discrete-time switched system with state delay:

$$\Sigma_0: \quad x_{k+1} = \sum_{i=1}^{N} \alpha_i(k)A_i x_k + \sum_{i=1}^{N} \alpha_i(k)A_{di}x_{k-d_k} + \sum_{i=1}^{N} \alpha_i(k)\Gamma_i\omega_k \tag{2.1}$$

$$y_k = \sum_{i=1}^{N} \alpha_i(k)C_i x_k + \sum_{i=1}^{N} \alpha_i(k)C_{di}x_{k-d_k} + \sum_{i=1}^{N} \alpha_i(k)\Phi_i\omega_k \tag{2.2}$$

$$z_k = \sum_{i=1}^{N} \alpha_i(k)G_i x_k + \sum_{i=1}^{N} \alpha_i(k)G_{di}x_{k-d_k} + \sum_{i=1}^{N} \alpha_i(k)\Psi_i\omega_k \tag{2.3}$$

where $x_k \in \Re^n$ is the state, $y_k \in \Re^r$ is the measured output, $z_k \in \Re^q$ is the signal to be estimated, $\omega_k \in \Re^p$ is the disturbance input which is assumed to belong to $l_2[0,\infty)$. The state delay d_k appearing in the hybrid system dynamics are frequently encountered in several system applications including networked control systems, chemical processes, population dynamics and economic systems [28]. In the sequel, it is assumed that d_k is time-varying and satisfying $\underline{d} \leqslant d_k \leqslant \bar{d}$, where the bounds $\underline{d}>0$ and $\bar{d}>0$ are constant scalars. The initial condition sequence $\{\alpha_k,\ k=-\bar{d},-\bar{d}+1,\ldots,0\}$ is given. In (2.1)–(2.3), $\alpha_i(k)$ is the switching signal,

$$\alpha_i : Z^+ \longrightarrow \{0,1\}, \quad \sum_{i=1}^{N} \alpha_i(k) = 1, \quad k \in Z^+ = \{0,1,\ldots\}$$

which specifies which subsystem will be activated at certain discrete time. The matrices A_i, A_{di}, B_i, Γ_i, C_i, C_{di}, D_i, Φ_i , G_i , G_{di}, F_i and Ψ_i are real and of compatible dimensions.

For a switching mode $i \in \mathbb{N}$, the associated matrices

$$\Omega_i := \begin{bmatrix} A_i & A_{di} & \Gamma_i \\ C_i & C_{di} & \Phi_i \\ G_i & G_{di} & \Psi_i \end{bmatrix} \tag{2.4}$$

contain uncertainties and they belong to a given real convex bounded polyhedral domain $\mathscr{D}_c$. Each uncertain matrix of this set can be written as an unknown convex combination of S_i given extreme matrices $\Omega_{1i},\Omega_{2i},\ldots,\Omega_{Si}$, that is, $\Omega_i \in \mathscr{D}_c$ if and only if

$$\Omega_i := \left\{ \sum_{p=1}^{S_i} \lambda_{ip}\Omega_{ip}, \sum_{p=1}^{M_i} \lambda_{ip} = 1,\ \lambda_{ik} \geqslant 0 \right\}, \quad \Omega_{ip} := \begin{bmatrix} A_{ip} & A_{dip} & \Gamma_{ip} \\ C_{ip} & C_{dip} & \Phi_{ip} \\ G_{ip} & G_{dip} & \Psi_{ip} \end{bmatrix} \tag{2.5}$$

where the S_0 vertices form a unit simplex and $\{A_0,\ldots,\Psi_0\}$ are known real constant matrices of appropriate dimensions which describe the ith nominal subsystem.

In this work, we seek to designing a switched filter of order s [1]described by

$$\Sigma_f: \quad \hat{x}_{k+1} = \sum_{i=1}^{N} \alpha_i(k)A_{fi}\hat{x}_k + \sum_{i=1}^{N} \alpha_i(k)B_{fi}y_k, \quad \hat{x}_0 = 0 \tag{2.6}$$

$$\hat{z}_k = \sum_{i=1}^{N} \alpha_i(k)C_{fi}\hat{x}_k + \sum_{i=1}^{N} \alpha_i(k)D_{fi}y_k \tag{2.7}$$

[1] $s=n$ stands for full-order filter and $1 \leqslant s \leqslant n$ signifies a reduced-order filter.