Digital Signal Processing 35 (2014) 14–20

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

Robust space–time adaptive processing with colored loading using iterative optimization

Shengqi Zhu *, Guisheng Liao *, Jingwei Xu, Yongchan Gao

National Laboratory of Radar Signal Processing, Xidian University, Xi'an Shaanxi 710071, China

ARTICLE INFO

Article history:
Available online 1 September 2014

Keywords:
Space–time adaptive processing (STAP)
Diagonal loading
Robust beamforming
Colored loading
Limited supported samples

ABSTRACT

The performance of space–time adaptive processing (STAP) may degrade dramatically if some undesired mismatches exist in real scenarios, such as array calibration error, distorted antenna shape, direction of arrival (DOA) and Doppler frequency mismatches between the actual and presumed responses to the desired target signal, insufficient training data samples and so on. In this paper, we develop a new approach to STAP that is robust to different variations in real scenarios. This method is based on the iterative optimization for the spatial–temporal separate filter. It is confirmed that this method belongs to the class of colored loading algorithms. The loading factor can be efficiently calculated based on the known level of the uncertainty mismatch sets of spatial temporal steering vectors. Computer simulations demonstrate that the proposed robust two-dimensional (2-D) beamformer with colored loading has attained better performance as compared to the conventional STAP algorithm.

1. Introduction

In recent years, adaptive array beamforming has been widely used in radar, sonar, wireless communications and other areas [1–5]. When the desired array steering vector is known accurately, adaptive array beamforming has superior performance on interference-plus-noise suppression. However, if some undesired mismatches exist in the environment, such as look direction error, array calibration error, distorted antenna shape, source local scattering and so on, it is known that the performance of adaptive beamforming will degrade dramatically. For one dimensional spatial beamforming, some effective methods have been presented to overcome the problem of the undesired mismatches. These methods include the robust Capon beamforming (RCB) [8], doubly constrained robust Capon beamformer (DCRCB) [9] and worst case optimization [10], which are widely applied in robust array signal processing [6–10].

Space–time adaptive processing (STAP) is a powerful tool for clutter suppression with a moving platform and therefore has been widely used in airborne radar [11–16]. However, it is well known that the performance of STAP may degrade severely in the presence of mismatches between the actual and presumed array responses to the desired signal when the supported samples are limited [17,18]. Diagonal loading [19] is an effective solution to mitigate these deleterious factors in practice, which is called robust STAP. Robust STAP uses diagonal loading technique to prevent high sidelobes and distorted main beams caused by the limited training samples, signal mismatch and non-stationary interference. The essence of robust STAP is to utilize the robust adaptive array beamforming [6–10] in space and time domain jointly. As the coupling exists in the spatial–temporal domain, conventional diagonal loading methods for STAP are *ad hoc* and cannot obtain loading factors in space and time domain separately. In addition, the mismatch information is different in space and time domain (i.e., the Doppler frequency mismatch is different from the array antenna mismatch). Loading factors for space and time domain will be different. Therefore, we should deal with the space and time information independently.

In this paper, we consider the mismatch information of space and time separately, and propose a new technique called the colored loading approach that augments the sample covariance matrix of STAP processor in a manner similar to that of traditional diagonal loading. This technique is a generalization of diagonal loading in which the covariance matrix is augmented with a colored matrix as opposed to using the identity matrix. The attractiveness of this technique is that loading factors for space and time domain are separated and can be obtained by the respective iterative optimization. This method has better performance compared with conventional diagonal loading STAP processor. In addition, these loading factors for either spatial or temporal domain can be precisely calculated by uncertainty mismatch sets. Numerical simulations show that the colored loading approach

* Corresponding authors.
E-mail addresses: zhushengqi8@163.com (S. Zhu), liaogs@xidian.edu.cn (G. Liao).

http://dx.doi.org/10.1016/j.dsp.2014.08.009
1051-2004/

is superior to conventional STAP in terms of the optimal output signal-to-interference plus noise ratio (SINR) in the snapshot deficient scenario.

This paper is organized as follows. In Section 2, we first give the signal model of STAP, and then propose the robust space–time 2-D filter. Performance analysis and relationship to other robust methods are given in Section 3, including the relationship to the LSMI beamformer and the knowledge-aided STAP. In Section 4, simulated data is employed to illustrate the effectiveness of the proposed scheme. Finally, conclusions are given in Section 5.

2. Problem formulation

2.1. Space–time signal model

The received data with a signal for STAP is $\mathbf{x}_{receive} = \alpha\mathbf{s} + \mathbf{c}$, where α denotes the known amplitude of the signal, $\mathbf{s} = \mathbf{a} \otimes \mathbf{b}$ denotes the spatial–temporal steering vector, and $\mathbf{c}$ denotes clutter and noise. In $\mathbf{s}$, $\mathbf{a}$ denotes the spatial steering vector, $\mathbf{b}$ denotes the temporal steering vector, and $\otimes$ denotes the Kronecker operator. Note that the data vector $\mathbf{x}_{receive}$ for a range bin can be rearranged as the following space–time data matrix $\mathbf{X}$

$$\mathbf{X} = \begin{bmatrix} x(1,1) & x(1,2) & \cdots & x(1,M) \\ x(2,1) & x(2,2) & \cdots & x(2,M) \\ \vdots & \vdots & \ddots & \vdots \\ x(N,1) & x(N,2) & \cdots & x(N,M) \end{bmatrix} \quad (1)$$

where N, M denote the number of spatial channel, temporal channel. In (2), each element $x(n,k)$, $n = 1, 2, \cdots, N$; $k = 1, 2, \cdots, M$ denotes the received data of the nth element and the kth pulse.

The optimum STAP data vector deals with space–time steering vector $\mathbf{s}$ so that the conventional STAP is essentially a spatial temporal inseparable filter. The optimum STAP is given by [20]

$$\min E\left\{\left\|\mathbf{w}^H \mathbf{x}_{receive}\right\|^2\right\} \quad \text{s.t. } \mathbf{w}^H\mathbf{s} = 1 \quad (2)$$

where $E\{\cdot\}$ denotes the expectation operator. The $NM \times 1$ weight vector $\mathbf{w}$ is given by $\mathbf{w} = \mathbf{R}^{-1}\mathbf{s}$. Here, the $NM \times NM$ space–time matrix $\mathbf{R}$ is the true covariance matrix. In practice, $\mathbf{R}$ is unknown and should be replaced by the $NM \times NM$ space–time sample covariance matrix $\hat{\mathbf{R}}$. Therefore, the weight with sample covariance matrix is the well known sample matrix inversion (SMI) approach for STAP [20].

Due to the fact that space and time error information is different, the space and time vectors can be dealt with separately. Therefore, the space–time adaptive weight matrix $\mathbf{W}$ can be written as follows

$$\mathbf{W} = \mathbf{u}\mathbf{v}^T \quad (3)$$

where $\mathbf{u}$ is the $N \times 1$ spatial weight vector and $\mathbf{v}$ is the $M \times 1$ temporal weight vector. The spatial temporal separable filter (STSF), i.e., is formulated as [21]

$$\min_{\mathbf{u},\mathbf{v}} E\left\{\left\|\mathbf{u}^H \mathbf{X}\mathbf{v}^*\right\|^2\right\} \quad \text{s.t. } \mathbf{u}^H\mathbf{a} = 1 \text{ and } \mathbf{v}^H\mathbf{b} = 1 \quad (4)$$

Formula (4) has two separate constraints: the spatial steering vector constraint $\mathbf{u}^H\mathbf{a} = 1$, and the temporal steering vector constraint $\mathbf{v}^H\mathbf{b} = 1$. However, the STSF is not the optimal space–time 2-D filter. It is a dimensional reduced 2-D filter. However, compared with traditional non-separable STAP with insufficient data samples, the performance of the STSF is shown better [21].

In addition, the one-dimensional worst case beamforming [10] can be applied to two-dimensional STAP. That is:

$$\min_{\mathbf{w}} \mathbf{w}^H\mathbf{R}\mathbf{w} \quad \text{s.t. } \left|\mathbf{w}^H\mathbf{s} - 1\right|^2 = \varepsilon^2\mathbf{w}^H\mathbf{w} \quad (5)$$

where $\mathbf{w}$ is the $NM \times 1$ weight vector, $\hat{\mathbf{R}}$ is the $NM \times NM$ space–time sample covariance matrix, ε is a known constant to bound the norm of the spatial steering vector distortion.

2.2. Robust space–time adaptive processing

The traditional robust STAP treats diagonal loading factors in space and time 2-D domain. However, we consider repeating the robust spatial and temporal processing separately by iterative optimization. Under worst case optimization, we get the following robust space–time 2-D filter

$$\min_{\mathbf{u},\mathbf{v}} E\left\{\left\|\mathbf{u}^H \mathbf{X}\mathbf{v}^*\right\|^2\right\} \quad \text{s.t. } \left|\mathbf{u}^H\mathbf{a} - 1\right|^2 = \varepsilon_1^2\mathbf{u}^H\mathbf{u}$$
$$\left|\mathbf{v}^H\mathbf{b} - 1\right|^2 = \varepsilon_2^2\mathbf{v}^H\mathbf{v} \quad (6)$$

The robust 2-D beamformer is essentially a spatial temporal separable filter. The formula above has two separate constraints: the robust spatial steering vector constraint $|\mathbf{u}^H\mathbf{a} - 1|^2 = \varepsilon_1^2\mathbf{u}^H\mathbf{u}$ and the robust temporal steering vector constraint $|\mathbf{v}^H\mathbf{b} - 1|^2 = \varepsilon_2^2\mathbf{v}^H\mathbf{v}$. The problem (6) includes $N + M$ independent variables (or DoF), which is lower than that of the joint space–time steering vector constraint $|\mathbf{w}^H\mathbf{s} - 1|^2 = \varepsilon^2\mathbf{w}^H\mathbf{w}$ (NM independent variables). We formulate a space–time 2-D filter and propose an effective approach to obtain loading factors in space and time domain.

Since the coupling between spatial and temporal processing exists, we should repeat the robust spatial and temporal processing iteratively until the convergence is achieved. To solve the problem in (6), we utilize the Lagrange multiplier method. Thereby, the cost function without constraint can be expressed as

$$L(\mathbf{u},\mathbf{v},\lambda_1,\lambda_2) = E\left\{\left\|\mathbf{u}^H\mathbf{X}\mathbf{v}^*\right\|^2\right\}$$
$$+ \lambda_1\left(\varepsilon_1^2\mathbf{u}^H\mathbf{u} - \mathbf{u}^H\mathbf{a}\mathbf{a}^H\mathbf{u} + \mathbf{u}^H\mathbf{a} + \mathbf{a}^H\mathbf{u} - 1\right)$$
$$+ \lambda_2\left(\varepsilon_2^2\mathbf{v}^H\mathbf{v} - \mathbf{v}^H\mathbf{b}\mathbf{b}^H\mathbf{v} + \mathbf{v}^H\mathbf{b} + \mathbf{b}^H\mathbf{v} - 1\right) \quad (7)$$

where λ_1 and λ_2 are Lagrange multipliers. We can use the conjugate gradient method or Newton method to obtain the optimal value. Due to the fact that the coupling exists between space and time domain, we use the bi-iterative algorithm [21,22] to calculate the robust spatial and temporal weights iteratively. Loading factors for space and time domain are separated and then adaptive weights of space and time domain can be updated by iteration processing in the presence of coupling. In this way, we can separately perform diagonal loading for spatial and temporal covariance matrices.

In the proposed algorithm, we separate spatial/temporal weight and the corresponding steering vectors based on the worst-case optimization or the RCB method [8], and form the spatial temporal separable filter (6), then we utilize the Lagrange multiplier method that is widely used in beamforming to solve the proposed algorithm (6). Finally, we use the bi-iterative algorithm to calculate the robust spatial and temporal weights iteratively. The joint space–time weight and steering vector can be obtained by the Kronecker product of the spatial/temporal weight and the corresponding steering vectors. Unlike conventional robust STAP with diagonal loading, we treat the space and time information independently and use λ_1 and λ_2 Lagrange multipliers to control the loading factor for space and time domains. The detailed procedures can be summarized as follows:

Step 1: At $k = 0$, initialize $\mathbf{v}(0) = \frac{\mathbf{b}}{\mathbf{b}^H\mathbf{b}}$, and calculate the differentiation of $L(\mathbf{u},\mathbf{v},\lambda_1,\lambda_2)$ with respect to $\mathbf{u}$ to zero. After some simple calculations, we can get the adaptive spatial weight vector $\mathbf{u}(k) = \frac{\lambda_1(k)}{\lambda_1(k)\mathbf{a}^H(\widehat{\mathbf{R}}_s(k)+\lambda_1(k)\varepsilon_1^2\mathbf{I})^{-1}\mathbf{a}-1}(\widehat{\mathbf{R}}_s(k) + \lambda_1(k)\varepsilon_1^2\mathbf{I})^{-1}\mathbf{a}$, where the spatial covariance matrix is $\widehat{\mathbf{R}}_s(k) = \frac{1}{P}\sum_{i=1}^{P}\mathbf{X}_i\mathbf{v}(k-1)\mathbf{v}^H(k-1)\mathbf{X}_i^H$, P is number of data samples, and the loading factor $\lambda_1(k)$