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A noise resistant image matching method using angular radial transform

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ABSTRACT

In this paper, we extend the concept of the optimal similarity measure, originally developed for Zernike moments (ZMs) which belong to a class of orthogonal rotation invariant moments (ORIMs), to angular radial transform (ART) which is non-orthogonal. The proposed distance measure not only uses the magnitude of the ART coefficients but also incorporates phase component unlike the existing L_1*-distance* and L_2*-distance* measures which use only the magnitude of ART in image matching problems. Experimental results show that the new distance measure outperforms L_2*-distance* measure. The performance of the proposed method is highly robust to Gaussian noise and salt-and-pepper noise even at very high level of noise. The results are compared with the ZMs-based optimal similarity measure. It is shown that the recognition rate of the proposed distance measure is comparable to that of the ZMs, however, at very low computational complexity.

1. Introduction

Angular radial transform (ART) [1] is a region-based shape descriptor among several rotation invariant descriptors such as Zernike moments (ZMs) [2], pseudo-Zernike moments (PZMs) [3], orthogonal Fourier-Mellin moments (OFMMs) [4], polar harmonic transforms (PHTs) [5], and several other rotation invariant moments and transforms (RIMTs) [6–8]. Although it is non-orthogonal unlike the other stated moments and transforms, it has two major advantages over others: its low computation complexity and better numerical stability. In addition to providing rotation invariance, it can be made translation and scale invariant after some geometric transformation. Being a region-based shape descriptor, it provides noise resilience capability. Due to its several attractive characteristics, MPEG-7 has adopted it as a region-based shape descriptor for image retrieval [9]. Also, these properties of ART have led to its wide applications in many image processing and pattern recognition domains, such as shape retrieval [10], video security systems [11], logo recognition system [12], and image watermarking [13]. In order to reduce its computational complexity, fast algorithms [14,15], have been developed to make it most suited for several real time applications where rotation invariant global shape descriptors are required.

A shape matching or image retrieval problem requires an effective similarity measure for its good performance. The existing similarity measures for the RIMTs features are based on the magnitude of the moments because they are rotation invariants. Normally, L_1*-distance*, L_2*-distance* (*Euclidean distance*), *Chi-square distance* and other similarity measures are used for the classification purpose. These are very fast and simple to use. However, the magnitude based similarity measures do not involve the phase component because it is not rotation invariant. It has been observed by Oppenheim and Lim [16] and Shao and Celenk [17] that the phase information is more effective than the magnitude of a signal in shape representation. Keeping this observation in view, many attempts have been made to incorporate phase information in representing features for pattern matching applications [18–21]. In particular, the approach developed by Revaud et al. [18] is very effective as compared to the classical and new similarity measures. In their approach, a new similarity measure, called optimal similarity measure, involving ZMs of two images being matched is obtained. This is based on the minimisation process of an error function representing distance between the reconstructed signals of the two images. Although the theory developed for the optimal similarity measure is based on the orthogonality of moments, we show that the method can be extended to non-orthogonal moments as well. In this paper, we derive an improved distance measure which involves both the magnitude and phase of ART coefficients following the concepts contained in the optimal similarity measure. The proposed distance measure is sub-optimal because it is not based on the orthogonality property of image signals. Motivated by the high recognition performance of optimal similarity measure for ZMs and the attractive features of the ART, the performance of the improved

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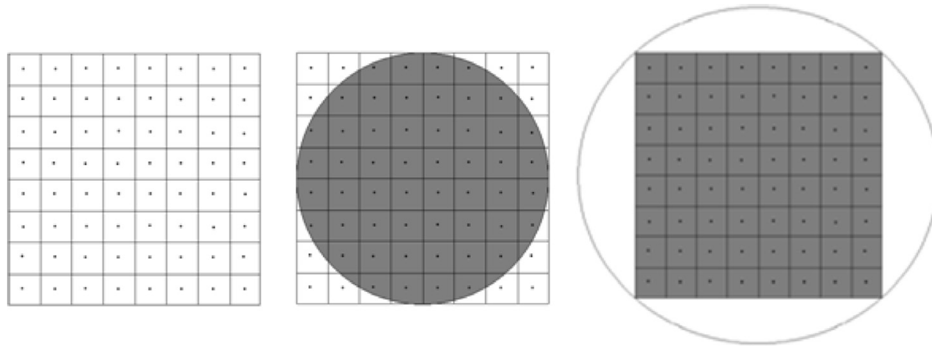

(a) (b) (c)

Fig. 1. (a) An 8 × 8 image grid, (b) inner unit disk mapping, and (c) outer unit disk mapping.

distance measure of the ART is compared with the commonly used classical L_2-*distance* measure. The performance of the new distance measure is also compared with ZMs features using optimal similarity measure for which the algorithm was originally developed. The performance of the proposed distance measure is observed to provide excellent recognition rates on noisy images corrupted by white Gaussian noise (on grayscale images) and salt-and-pepper noise (on binary images). Detailed experiments are performed to analyse the performance of ART and compare its results with ZMs. A note on its extension to color images is also presented.

The rest of the paper is organised as follows. An overview of ART is discussed in Section 2. The optimal similarity measure for ART is developed in Section 3. Detailed experimental analysis is carried out in Section 4. Section 5 presents conclusion.

2. The angular radial transform (ART)

The ART of a function $f(r,\theta)$ of order n and repetition m on a unit disk is defined by [1]

$$A_{nm} = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 f(r,\theta)V_{nm}^*(r,\theta)rdrd\theta, \tag{1}$$

where n is a non-negative integer, m is an integer, $V_{nm}^*(r,\theta)$ is the complex conjugate of the kernel function $V_{nm}(r,\theta)$ which itself is defined by

$$V_{nm}(r,\theta) = R_n(r)\omega_m(\theta). \tag{2}$$

The kernel functions $V_{nm}(r,\theta)$ are separable into radial function $R_n(r)$ and angular functions $\omega_m(\theta)$ with

$$\omega_m(\theta) = e^{jm\theta}, \tag{3}$$

where $j=\sqrt{-1}$. The radial functions $R_n(r)$ are defined by

$$R_n(r) = \begin{cases} 1, & \text{if } n=0 \\ 2\cos(\pi nr), & \text{otherwise.} \end{cases} \tag{4}$$

We observe that the angular functions $\omega_m(\theta)$ are orthogonal while the radial functions are not orthogonal, because

$$\int_0^{2\pi} \omega_m(\theta)\omega_{m'}^*(\theta)d\theta = 2\pi\delta_{mm'} \tag{5}$$

and

$$\int_0^1 R_n(r)R_{n'}^*(r)rdr = \begin{cases} \frac{1}{4}, & \text{if } n=n' \\ \frac{1}{2\pi^2}\left[\frac{(-1)^{n+n'}-1}{(n+n')^2} + \frac{(-1)^{n-n'}-1}{(n-n')^2}\right], & \text{otherwise,} \end{cases} \tag{6}$$

where $\delta_{mm'}=1$, if $m=m'$ and 0, otherwise.

ART for digital images: The ART coefficients defined by Eq. (1) pertain to a continuous signal. On the other hand, the image signals are digital and defined over a rectangular domain $M \times N$ with M rows and N columns. For simplicity we take a square image of size $N \times N$. Since the transform coefficients are computed on a unit disk, the following mapping converts the digital domain into a unit disk:

$$x_i = \frac{2i+1-N}{D}, \qquad y_k = \frac{2k+1-N}{D}, \qquad i,k = 0,1,\ldots,N-1 \tag{7}$$

where

$$D = \begin{cases} N, & \text{for inner unit disk} \\ N\sqrt{2}, & \text{for outer unit disk} \end{cases}$$

The coordinate of the centre of the pixel (i,k) is given by (x_i, y_k) which occupies the area

$$\left[x_i - \frac{\Delta x}{2}, x_i + \frac{\Delta x}{2}\right] \times \left[y_k - \frac{\Delta y}{2}, y_k + \frac{\Delta y}{2}\right], \quad \text{where}$$
$$\Delta x = \Delta y = \frac{2}{D}. \tag{8}$$

A choice of D depends whether one uses inner unit disk ($D=N$) or the outer unit disk ($D=N\sqrt{2}$). Figs. 1(a), 1(b) and 1(c) show an 8 × 8 pixel grid, the inner unit disk mapping and the outer unit disk mapping, respectively. In many pattern recognition problems it is observed that the outer unit disk mapping provides better performance than the inner unit disk mapping [22,23]. In our experiments, we have not observed much difference in the performance between the two mappings, therefore, we use inner unit disk mapping in all our experimental setups.

Since it is difficult to find an analytical solution to the double integration involved in Eq. (1), the zeroth order approximation is generally used to find transform coefficients after converting the polar form of Eq. (1) into its Cartesian equivalent, i.e.