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Exact conditional and unconditional Cramér–Rao bounds for near field localization

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ABSTRACT

This paper considers the Cramér–Rao lower Bound (CRB) for the source localization problem in the near field. More specifically, we use the exact expression of the delay parameter for the CRB derivation and show how this 'exact CRB' can be significantly different from the one given in the literature based on an approximate time delay expression (usually considered in the Fresnel region). In addition, we consider the exact expression of the received power profile (i.e., variable gain case) which, to the best of our knowledge, has been ignored in the literature. Finally, we exploit the CRB expression to introduce the new concept of Near Field Localization Region (NFLR) for a target localization performance associated to the application at hand. We illustrate the usefulness of the proposed CRB derivation as well as the NFLR concept through numerical simulations in different scenarios.

1. Introduction

Sources localization problem has been extensively studied in the literature but most of the research works are dedicated to the far field case, e.g., [1,2].

In this paper, we focus on the situation where the sources are in a near field (NF) region which occurs when the source ranges to the array are not 'sufficiently large' compared with the aperture of the array system [3,4]. Indeed, this particular case has several practical applications including speaker localization and robot navigation [5,6], underwater source localization [7], near field antenna measurements [8,9] and certain biomedical applications, e.g., [10]. Many localization methods have been dedicated to this problem ranging from maximum likelihood to subspace methods, e.g. [11–13] and using different signal features, e.g. cyclostationarity [14], sparsity [15] or non-Gaussianity [4]. Recently, some works considered both far field and near field localization [16,17] where the authors propose different methods to achieve a better localization performance when the source moves from far field to near field and vice versa.

More specifically, this paper is dedicated to the derivation of the Cramér–Rao Bound (CRB) expressions for different signal models and their use for the better understanding of this particular localization problem. CRB derivation for the near field case has already been considered in the literature [3,18–21]. In [3,18], the exact expression of the time delay has been used to derive the unconditional CRB in matrix form, i.e., expressed and computed numerically as the inverse of the Fisher Information Matrix (FIM). In [19], the conditional CRB based on an approximate model (i.e., approximate time delay expression as shown in Section 2.1) is provided.

Recently, El Korso et al. derived analytical expressions of the conditional and unconditional CRB for near field localization based on an approximate model [20]. In the latter work, both conditional and unconditional CRB of the angle parameter are found independent from the range value and are equal to those of far field region. This CRB derivation has been extended in [21] to non-ideal antenna arrays with unknown phase and gain responses.

In our work, we propose first, to use the exact time delay expression as well as the exact power profile (i.e., Variable Gain (VG) model) corresponding to the spherical form of the wavefront for the derivation of closed form formulas of the conditional and unconditional VG-CRBs. Indeed, in the near field case, the received power is variable from sensor to sensor which should be taken into account in the data model. By considering such variable gain model, we investigate the impact of the gain variation onto the localization performance limit.

Secondly, we consider a simple case where the sensor to sensor power variation is neglected[1] (i.e., Equal Gain (EG) model).

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[1] To the best of our knowledge, this assumption is considered in all previous works.

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The obtained EG-CRB expressions using the exact time delay values are then compared to those given in [20]. The development (i.e., Taylor expansion) of the exact EG-CRB allows us to highlight many interesting features including:

- A more accurate approximate EG-CRB for the exact model as compared to the EG-CRB based on an approximate model. In particular, we show that at low range values, the approximate EG-CRB in [20] can be up to 30 times larger than our exact EG-CRB.
- A detailed analysis of the source range parameter effect on its angle estimation performance.[2]

Finally, we propose to exploit the exact CRB expression to specify the 'near field localization region' based on a desired localization performance. In that case, the 'near field localization region' is shown to depend not only on the source range parameter and array aperture but also on the sources SNR and observation sample size.

The paper is organized as follows: Section 2 introduces the data model and formulates the main paper objectives. In Section 3, the conditional and unconditional VG-CRB derivations are provided in the variable gain case. Section 4 presents the simple case where all sensors have the same gain (i.e., EG case). The EG-CRB is then compared to the VG-CRB with an analysis of the impact of the gain profile onto the localization performance. In addition, we show in Section 4.2 that considering the exact time delay expression leads to a more accurate CRB expression as compared to the CRB based on an approximate time delay. Section 5 introduces the concept of near field localization region and illustrates its usefulness through specific examples. Section 6 is dedicated to simulation experiments while Section 7 is for the concluding remarks.

2. Problem formulation

2.1. Data model

In this paper, we consider a uniform linear array with N sensors receiving a signal, emitted from one source located in the NF region, and corrupted by circular white Gaussian noise v_n of covariance matrix $\sigma^2\mathbf{I}_N$. The nth array output, $n = 0, \cdots, N-1$, is expressed as

$$x_n(t) = \gamma_n(\theta, r)s(t)e^{j\tau_n(\theta,r)} + v_n(t)$$
$$= s(t)\big(\gamma_n(\theta, r)e^{j\tau_n(\theta,r)}\big) + v_n(t) \quad t = 1, \cdots, T, \tag{1}$$

where T is the sample size and $\gamma_n(\theta, r)$ represents the power profile of the nth sensor given by [23]

$$\gamma_n(\theta, r) = \frac{1}{l_n} = \frac{1}{r\sqrt{1 - \frac{2nd}{r}\sin\theta + (\frac{nd}{r})^2}}, \tag{2}$$

l_n being the distance between the source signal and the nth sensor and $s(t)$ is the emitted signal. The exact expression of the time delay[3] τ_n is given by

$$\tau_n = \frac{2\pi r}{\lambda}\left(\sqrt{1 + \frac{n^2d^2}{r^2} - \frac{2nd\sin\theta}{r}} - 1\right), \tag{3}$$

where d is the inter-element spacing, λ is the propagation wavelength and (r, θ) are the polar coordinates of the source as shown in Fig. 1.

[2] Part of the work related to the EG-CRB has been published in [22] presented at conference ISSPA 2012.

[3] The first sensor, $n = 0$, is considered for the time reference.

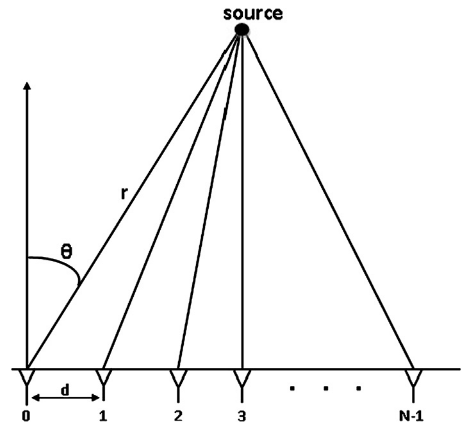

Fig. 1. Near field source model.

In the sequel, the source signal will be treated either as deterministic (conditional model) or stochastic (unconditional model). Indeed, in the array processing, both models can be found:

1) Conditional model in which the source signal is assumed deterministic but its parameters are unknown.
2) Unconditional model in which we assume that the source signal is random. In our case, we will assume $s(t)$ to be a complex circular Gaussian process with zero mean and unknown variance σ_s^2. Note that all existing works on the near field CRB consider such an assumption. Indeed, in the Gaussian case, the CRB derivation is tractable and 'interpretable' closed form expressions can be obtained. In addition, as shown in [24] and more recently in [25] for the conditional model, the Gaussian case is the least favorable one and hence it represents the most conservative choice. In other words, any optimization based on the CRB under the Gaussian assumption can be considered to be min–max optimal in the sense of minimizing the largest CRB.

Remark. Note that, the more general case of multipath channel is not considered here for the 'tractability' of the CRB derivation and its 'readability'. Indeed, in most existing papers, e.g. [17], one associates to each source one direction of arrival (i.e., one channel path). The source localization in multipath case can be treated as a multiple correlated sources case as in [26].

2.2. Objectives

In the literature, only approximate models of the received NF signals are considered. Indeed, two simplifications of the model in (1) are used in practice:

(a) The sensor to sensor power variation is neglected and the power profile is approximated by $\gamma_n(\theta, r) = \frac{1}{r}$. This model is referred to as the equal gain model since $\gamma_n(\theta, r)$ is constant with respect to the sensor index n.

(b) Also, most existing works on near field source localization consider the following approximation of the time delay expression to derive simple localization algorithms as well as CRB expressions, e.g., [27,20]

$$\tau_n = -2\pi\frac{dn}{\lambda}\sin(\theta) + \pi\frac{d^2n^2}{\lambda}\frac{1}{r}\cos^2(\theta) + o\left(\frac{d}{r}\right). \tag{4}$$

The latter is considered as a good approximation of (3) in the Fresnel region given by [23]