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Shrinkage estimation-based source localization with minimum mean squared error criterion and minimum bias criterion

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ABSTRACT

In this paper, we propose two novel source localization methods; one is the shrinkage estimator with the minimum mean squared error criterion, and the other is the shrinkage estimator with the minimum bias criterion. The mean squared error performance of the two-step weighted least squares deteriorates in the large noise variance regimes. In order to improve the two-step weighted least squares in the large noise variance regimes, the shrinkage factor is multiplied by the two-step weighted least squares estimator, and then the novel estimator is determined such that the mean squared error and squared bias are minimized. Simulation results show that the mean squared error performances of the proposed methods are better than those of the two-step weighted least squares method as well as the minimax estimator in a regime with large measurement noise variances.

1. Introduction

Source localization is a technique that finds a geometrical point of intersection using the measurements from each receiver, such as the time difference of arrival, the time of arrival (TOA), or the received signal strength. Localizing point sources, using passive and stationary sensors, is of considerable interest, and this has been a repeated theme of research in the radar, sonar, global positioning system, video conferencing, and telecommunication areas.

Several methods exist for multi-sensor localization. Torrieri [1] derived a principal algorithm to analyze the hyperbolic location systems and direction-finding location systems and used distance and angle information for the maximum likelihood (ML). In [2], the source position was estimated using the adaptive filter theory presented therein. Performance analysis of source localization methods are included in the research results of [3] and [4]. The accuracies of least squares (LS) and squared-range LS (SRLS) were compared in [3]. The researchers in [4] discussed whether the three source localization methods are minimum variance unbiased estimator (MVUE) and robust. The minimax estimator, where the shrinkage factor was adopted, that dominates the LS estimator was developed in [5]. A closed-form solution satisfying the Cramér–Rao lower bound (CRLB) in a sufficiently small noise variance regime was developed in [6,7]. This method considered the relationship between the source position variable and the auxiliary range variable in order to calculate the location estimate in the second step. In [8], various shrinkage estimators were introduced. The shrinkage estimator is a type of biased estimator, e.g., the James–Stein estimator [9] and ridge regression [10], and is known for exhibiting strong optimality [11]. The minimum mean squared error (MSE) criterion has been widely used for optimal MSE performance of an estimator [11]. The minimum bias criterion has also been used in statistical estimation fields [12–15]. In the proposed methods, the shrinkage factor is multiplied by the existing two-step weighted least squares (WLS) method, and this constant is determined such that the MSE and the bias are minimized. These methods are different from the minimax estimator [5, Eqs. (44), (45)], because the shrinkage factors in these methods are different from that in the minimax estimator. The closed-form solution has advantages in terms of the number of measurements required in the determination of the position estimate and computational complexity, when compared with an iteration method, such as the Taylor-series ML, particle filter, or gradient-based methods. The closed-form location estimation solution, which approximates the CRLB under a sufficiently small measurement noise condition, was developed in [6,7]. In situations where variance of the measurement noise is sufficiently small, the second-order noise terms can be neglected.

However, the performances of the existing methods [6,7] degrade when the variance of the measurement noise is large, because the second-order noise components cannot be neglected. These second-order noise terms render the mean of the noise components [defined as $E[\mathbf{w}]$, $E[\mathbf{v}]$ in Eqs. (15), (22)] to be nonzero under large noise conditions, and the conventional minimax estimator can be improved by correcting the shrinkage factor using

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this phenomenon. In the two-step WLS method, the second-order noise terms increase the bias under large noise conditions and thereby increase the MSE. This paper proposes methods that use the shrinkage estimator with the minimum MSE and minimum bias, respectively. The proposed methods outperform the two-step WLS method [6,7] as well as the minimax estimator for large noise conditions.

The organization of this paper is as follows. Section 2 deals with the details of the existing two-step WLS source localization algorithm and discusses the proposed methods. Section 3 performs the MSE analysis of the proposed method. Section 4 analyzes the experiment results to evaluate the estimation performance of the proposed methods, comparing it with existing algorithms. Section 5 presents conclusions and directions for future work.

2. Problem formulation

The TOA source localization method finds the position of a source by using multiple circles whose centers represent the positions of sensors.

The measurement equation is represented as

$$r_{k,i} = r_i^o + n_{k,i} = \sqrt{(x-x_i)^2 + (y-y_i)^2} + n_{k,i}, \quad i = 1, 2, \dots, M,\ k = 1, 2, \dots, N \tag{1}$$

where $r_{k,i}$ and r_i^o are the measured and true distances, respectively, between the source and the ith sensor at the kth sampling, $n_{k,i}$ is the measurement noise, modeled as a Gaussian distribution and represented as $N(0, \sigma_i^2)$, $[x, y]^T$ is the true source position, and $[x_i, y_i]^T$ is the position of the ith sensor.

Throughout this letter, a lowercase boldface letter denotes a vector, an uppercase boldface letter indicates a matrix, and the superscript T signifies the vector/matrix transpose.

Ignoring the measurement noise and squaring (1) yields the following equations.

$$x_i x + y_i y - 0.5R = 0.5\left(x_i^2 + y_i^2 - r_{k,i}^2\right), \quad i = 1, 2, \dots, M,\ k = 1, 2, \dots, N \tag{2}$$

which is represented in a matrix form as

$$\mathbf{A}\mathbf{x} = \mathbf{b}_k, \tag{3}$$

where

$$\mathbf{A} = \begin{bmatrix} x_1 & y_1 & -0.5 \\ \vdots & \vdots & \vdots \\ x_M & y_M & -0.5 \end{bmatrix}, \qquad \mathbf{b}_k = \frac{1}{2}\begin{bmatrix} x_1^2 + y_1^2 - r_{k,1}^2 \\ \vdots \\ x_M^2 + y_M^2 - r_{k,M}^2 \end{bmatrix},$$

$$\mathbf{x} = [x \quad y \quad R]^T,$$

x_i, y_i is the position of the ith sensor, $r_{k,i}$ is the measured distance between the source and the ith sensor at the kth sampling, and R is the range variable defined as $x^2 + y^2$.

In this paper, the multiple measurements scenario ($N \geqslant 2$) is assumed, and the optimal estimator is just a sample mean for this case.

This can be shown as follows. The measurements $\mathbf{b}_k$ ($k = 1, \dots, N$) in (3) have an approximate Gaussian distribution as shown in (4).

$$\begin{aligned} b_{k,i} &= 0.5\left(x_i^2 + y_i^2 - r_{k,i}^2\right) \\ &= 0.5\left\{x_i^2 + y_i^2 - \left(r_i^o + n_{k,i}\right)^2\right\} \\ &= 0.5\left\{x_i^2 + y_i^2 - \left(r_i^o\right)^2 - 2r_i^o n_{k,i} - n_{k,i}^2\right\}, \end{aligned} \tag{4}$$

$$\mathbf{b}_k = [b_{k,1}, \dots, b_{k,M}]^T$$

where r_i^o is the true distance between the source and the ith sensor.

The second-order noise terms can be neglected in a sufficiently small noise condition in (4). Hence, $b_{k,i}$ ($k = 1, \dots, N$, $i = 1, \dots, M$) is an approximately Gaussian distribution.

The covariance matrix of $\mathbf{b}_k$ is obtained as follows:

The noise components of $\mathbf{b}_k$ are represented from (4) as follows when the second-order noise term is ignored:

$$\Delta b_{k,i} = -r_i^o n_{k,i} \tag{5}$$

The following property is obtained from (5) because $n_{k,i}$, $n_{k,j}$ ($i \neq j$) are uncorrelated.

$$E[\Delta b_{k,i} \cdot \Delta b_{k,j}] = \begin{cases} 0, & \text{if } i \neq j \\ (r_i^o)^2 \sigma_i^2, & \text{if } i = j \end{cases} \tag{6}$$

$$E[\Delta b_{k,i}] = 0, \qquad E[\Delta b_{k,j}] = 0 \quad \text{for all } k, i, \text{ and } j \tag{7}$$

Thus, the covariance matrix of $\mathbf{b}_k$ is obtained as follows:

$$\begin{aligned} [\mathbf{C}_b]_{i,j} &= E[\Delta b_{k,i} \cdot \Delta b_{k,j}] - E[\Delta b_{k,i}] \cdot E[\Delta b_{k,j}] \\ &= \begin{cases} (r_i^o)^2 \sigma_i^2, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases} \\ &\cong \begin{cases} r_i^s \sigma_i^2, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases} \end{aligned}$$

where $r_i^s = (1/N)\sum_{k=1}^{N} r_{k,i}^2$,

$[\mathbf{C}_b]_{i,j}$ denotes the component corresponding to the ith row and jth column of $\mathbf{C}_b$. (8)

In the derivation of $\mathbf{C}_b$, $(r_i^o)^2$ is substituted as r_i^s because it is unknown.

When assuming $r_{k,i}$ and $r_{m,i}$ ($k \neq m$) obtained from different time are independent, $b_{k,i}$, $b_{m,i}$ ($k \neq m$) are also independent. Then, the joint probability distribution of $\mathbf{b}_{1:N}$ can be represented as follows:

$$p(\mathbf{b}_{1:N}; \mathbf{x}) = (2\pi)^{-MN/2} |\mathbf{C}_b|^{-N/2} \times \exp\left\{-0.5 \sum_{k=1}^{N} (\mathbf{b}_k - \mathbf{A}\mathbf{x})^T \mathbf{C}_b^{-1} (\mathbf{b}_k - \mathbf{A}\mathbf{x})\right\} \tag{9}$$

Eq. (9) is the exponential family form, and the sufficient statistic of the exponential family is a complete sufficient statistic [16]. Hence, the sufficient statistic of (9) (i.e., $\mathbf{A}^T \mathbf{C}_b^{-1} \sum_{k=1}^{N} \mathbf{b}_k$) is the complete sufficient statistic. The first-step location estimate is obtained as (10) because the unbiased function of the complete sufficient statistic is the MVUE [16]. However, (10) is not an exact MVUE in the large noise variance condition, due to the second-order noise terms. Thus, it should be further improved.

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T \mathbf{C}_b^{-1} \mathbf{A}\right)^{-1} \mathbf{A}^T \mathbf{C}_b^{-1} \left\{(1/N) \sum_{k=1}^{N} \mathbf{b}_k\right\} \tag{10}$$

The relationship between the position and range variable is additionally used in the second-step to obtain a more accurate estimate.

The second-step position estimate is determined as follows [2]:

When $\hat{\mathbf{x}}$ of (10) is sufficiently close to $\mathbf{x}$, we have

$$\begin{aligned} [\hat{\mathbf{x}}]_1^2 - x^2 &= \left([\hat{\mathbf{x}}]_1 + x\right)\left([\hat{\mathbf{x}}]_1 - x\right) \cong 2x\left([\hat{\mathbf{x}}]_1 - x\right) \\ [\hat{\mathbf{x}}]_2^2 - y^2 &= \left([\hat{\mathbf{x}}]_2 + y\right)\left([\hat{\mathbf{x}}]_2 - y\right) \cong 2y\left([\hat{\mathbf{x}}]_2 - y\right) \end{aligned} \tag{11}$$

where $\hat{\mathbf{x}}$ was defined in (10), and x, y are the coordinates of the source.