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Improved Wigner–Ville distribution performance based on DCT/DFT harmonic wavelet transform and modified magnitude group delay

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Abstract

A new Wigner–Ville distribution (WVD) estimation is proposed. This improved and efficient WVD is based on signal decomposition (SD) by DCT or DFT harmonic wavelet transform (DCTHWT or DFTHWT) and the modified magnitude group delay (MMGD). The MMGD processing can be either in fullband or subband. The SD by DCTHWT provides better quality low leakage decimated subband components. The concatenation of WVDs of the subbands results in an overall WVD, significantly free from crossterms and Gibbs ripple. As no smoothing window is used for the instantaneous autocorrelation (IACR), MMGD removes or reduces the Gibbs ripple preserving the frequency resolution achieved by the DCT/DFT HWT. The SD by DCTHWT compared to that of DFTHWT, has improved frequency resolution and detectability. These are due to the symmetrical data extension and the consequential low leakage (bias and variance). As the zeros due to the associated white noise are removed by the MMGD effectively in subband domain than in fullband, the proposed WVD based on subband has a better noise immunity. Compared to fullband WVD, the subband WVD is computationally efficient and achieves a significantly better: frequency resolution, detectability of low-level signal in the presence of high-level one and variance. The SD-based methods, however cannot bring out the frequency transition path from band to band clearly, as there will be gap in the contour plot at the transition. For the proposed methods, the heart rate variability (HRV) real data is also considered as an example.

Keywords: Wigner–Ville distribution; DFT harmonic wavelet transform; DCT harmonic wavelet transform; Signal decomposition; Modified magnitude group delay function; Fullband and subband processing; Heart rate variability (HRV) data

1. Introduction

The Wigner–Ville distribution (WVD) is used for the analysis of nonstationary signals. In practice, the pseudo-WVD (PWVD), the Fourier transform of the instantaneous autocorrelation (IACR) computed only for a finite number of lags, is used. In the PWVD, to overcome the IACR truncation effect

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(Gibbs ripple), a smoothing *window function* is used. For a given lag length, the windowing *deteriorates the frequency resolution*. For a multicomponent signal, the WVD being quadratic in nature introduces undesired crossterms. The crossterms can be reduced by time smoothing *but only at the cost of time resolution.* To suppress crossterm effectively, to improve the frequency resolution and to maintain the desired time–frequency representation (TFR) properties, many distributions are proposed with varying degree of success. The prominent are: the Choi–William's, Cone–Kernel and reduced interference Kernel [1].

Recently, a WVD based on signal decomposition (SD) and modified magnitude group delay—MMGD (FBWVD), has been proposed [3]. The SD is realized by a perfect reconstruction filter bank (PRFB). The Gibbs ripple due to IACR truncation, manifests as zeros close to the unit circle. The MMGD removes these zeros without applying any smoothing window function [4]. The removal of zeros doest not disturb the signal pole locations and hence preserves the frequency resolution of a rectangular window.

The PRFB reduces the crossterms effect for a multicomponent signal, as the overall IACR is achieved by the summation of their individual IACRs computed separately [2]. However the SD by PRFB and computation of the individual IACRs at *the original sampling rate*, *are computationally intensive*. The decimation for the IACR, may appear to reduce computations. This is not so as WVD requires both decimation and interpolation operations, which by themselves are computationally intensive.

The DFT harmonic wavelet transform (DFTHWT) [5] can decompose and reconstruct the signal, without *directly* performing the decimation and interpolation operations. In DFTHWT, these operations are built in. As the decimated components are readily available in the frequency domain, certain type of processing, like group delay is directly applicable. This makes the overall algorithm simple and computationally efficient [9].

The DFTHWT is good as long as no processing of the components is involved prior to inverse transformation. It may also be tolerable for a signal with well-separated frequency components of high magnitude. To get improved WVD, after SD, it is required to process individual subband components differently. In such a case, decomposing the signal based on DFTHWT and processing the resulting subbands may not be very effective. This is because the signal energy from one component to another has *already leaked* during the FT computation.

As the DCT extends the signal symmetrically, it results in a significant reduction in abruptness of truncation and hence the leakage effect. This is due to smooth transition from one period to another (built-in periodicity) in DCT. It appears as if there is *no windowing and no side lobes to enhance the Gibbs and leakage effects* [10a]. It also improves the detection ability of a smaller magnitude component in the presence of a larger one. These facts motivated the authors to extend the desirable spectral properties of the DCT [10b] to harmonic wavelet transform (HWT) by grouping the DCT coefficients [11], instead of DFT coefficients.

This paper proposes a new efficient approach to improve the performance of WVD. This is based on the SD by DCT/DFTHWT and MMGD function. The SD by DCT/DFT HWT and the MMGD remove/reduce the existence of the crossterms and the Gibbs ripple due to truncation of the IACR, respectively. The application of MMGD to subband components improves both the frequency resolution and noise immunity compared to those of fullband. The SD by DCTHWT is advantageous over that by PRFB and DFTHWT as it results in reduction in computations, improved frequency resolution and signal detection ability.

2. Wigner–Ville distribution [1]

The WVD of an analytic signal $x(t)$is given by

$$W_x(t,\omega) = \int_{-\infty}^{\infty} r(t,\tau)\,\mathrm{e}^{-\mathrm{j}\omega\,\tau}\,\mathrm{d}\tau, \tag{1}$$

where $r(t,\ \tau) = x(t+\tau/2)\ \ x^{*}(t-\tau/2)$ is called the IACR function/Wigner kernel. From the computational point of view, the IACR can be considered only for a finite number of lags. This implies application of an inevitable rectangular window, which results in the Gibbs ripple effect. A smoothing window is applied to the IACR to reduce the Gibbs ripple and the resulting WVD is known as Pseudo-Wigner–Ville distribution (PWVD), given by

$$\mathrm{PW}_x(t,\omega) = \int_{-\infty}^{\infty} x(t+\tau/2)x^{*}(t-\tau/2)h^{2}(\tau/2)\,\mathrm{e}^{-\mathrm{j}\omega\tau}\,\mathrm{d}\tau. \tag{2}$$

The window function $h^2(\tau/2)$ reduces the Gibbs ripple. But it results in a reduction in frequency