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An efficient conjugate gradient method in the frequency domain: Application to channel equalization

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Abstract

In this paper, a new block adaptive filtering algorithm, based on the conjugate gradient (CG) method of optimization, is proposed. A Toeplitz approximation of the auto-correlation matrix is used for the estimation of the gradient vector and the involved quantities are updated on a block by block basis. Due to this formulation, the algorithm can be efficiently implemented in the frequency domain (FD). To this end, recursive relations for the FD quantities updated on a block by block basis have been derived. Different ways of accelerating convergence based either on the use of a preconditioner or on an appropriate decoupling of the direction vector are described. The applicability of the new algorithm to adaptive linear equalization (LE) and decision feedback equalization (DFE) has been studied. The proposed LE and DFE algorithms exhibit superior convergence properties as compared to existing adaptive algorithms, offering significant savings in computational complexity.

Keywords: Frequency domain adaptive filtering; Conjugate gradient method; Block adaptive DFE

1. Introduction

Adaptive filtering has been an area of active research over the last decades due to their wide applicability in many signal processing and communication applications. The performance of an adaptive algorithm can be assessed by a number of factors such as accuracy of steady state solution, convergence speed, tracking abilities, computational complexity, numerical robustness, etc. [1]. In many real time applications the issues of complexity and convergence speed play a crucial role, therefore many different techniques such as partial updating schemes [2], IIR adaptive filtering [3], and frequency domain (FD) adaptive filtering (e.g. [1,4]) have been developed to reduce computational complexity, and accelerate convergence.

The main idea in partial updating schemes is to reduce the computational complexity of an adaptive filter by adapting a group of the filter coefficients rather than the entire filter at every iteration. In non-stationary environments, however, partial updating adaptive filters might be undesirable as they do not guarantee convergence. The use of IIR adaptive filters dramatically reduces the computational complexity since a good performance can be achieved by estimating a small number of

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parameters. However, the stability of IIR schemes in certain implementation platforms is still an issue under investigation. Frequency domain adaptive filters (FDAF) turn out to be a good alternative in several practical applications due to their computational efficiency and their good convergence properties.

Most of the existing FDAF algorithms are of the gradient type, that is, their time-domain (TD) counterparts are based on some variations of the least mean square (LMS) algorithm. On the contrary, to the best of our knowledge, little work has been done toward developing FD implementations of adaptive algorithms based on the conjugate gradient (CG) method. CG methods were developed for iterative solutions of finite linear equations in the early 1950s [5,6]. Later, these methods were extended to solving non-linear equations or optimization problems [5,7]. The aim in CG methods is to accelerate the slow convergence rate of the steepest descent method while avoiding the involvement of a Hessian matrix associated with Newton methods. It is known that the CG methods terminate in at most M iterations, where M is the number of variables being adjusted, when they are applied to the minimization of a positive definite quadratic function [6].

Boray and Srinath proposed a CG algorithm for adaptive filtering applications including adaptive equalization, adaptive prediction and echo cancellation [8]. The convergence rate of their algorithm is comparable to that of the classical RLS algorithm. However, its computational burden is still high as compared with the LMS algorithm. Lim and Un suggested an algorithm for block adaptive filtering using the CG method [9]. More generally, the algorithms that are based on the CG method of optimization offer convergence comparable to RLS schemes at a computational cost that lies between the LMS and the RLS methods. In the existing literature, depending upon the CG algorithm under consideration the cost of sequential processing of the data grows sub-exponentially or quadratically with the filter order.

In this paper, a new block adaptive CG algorithm implemented in the FD is developed. In the resulting algorithm, one CG iteration per block update is executed. To reduce the complexity of the algorithm, a Toeplitz approximation of the autocorrelation matrix is used. Thus the involved matrix–vector products are calculated efficiently by employing the FFT algorithm. To avoid the convergence degradation of the proposed scheme, due to the above approximations, two different methods of accelerating convergence could be applied. The first method is based on the use of a preconditioner and the second one on an appropriate decoupling of the direction vectors.

The idea of applying the preconditioned CG (PCG) method in block adaptive filtering has also been proposed in [10,11]. In the technique proposed there, the classical PCG method was applied for solving a Toeplitz system of equations that had resulted after applying a proper data windowing [1]. According to that technique, the PCG process is repeated for each incoming block of input data and the filter parameters at each block are initialized to the optimal values determined from the preceding one. However, in the proposed scheme here, only one iteration of the PCG method per incoming block of data is executed. The second method of accelerating convergence is equivalent to the use of matrix step sizes instead of scalar ones which has been successfully applied in other FD adaptive filtering algorithms such us FDLMS [1,4].

In the proposed technique, a method of the second type (i.e., decoupling of the successive direction vectors) has been applied and the convergence of the resulting algorithm has become comparable to existing CG adaptive algorithms [12]. The application of the algorithm to channel equalization, for both the linear and the decision feedback equalization case, is also pointed out. The inherent causality problem appearing in the block formulation of the DFE is overcome by replacing the unknown decisions with properly derived tentative decisions [13]. The resulting algorithms exhibit convergence, comparable with the RLS algorithm and a computational complexity that is proportional to the logarithm of the filter order M per time step as compare to the $O(M^2)$, $O(M)$ and $O(\log(M))$ complexity of RLS, LMS and FD block LMS, respectively.

The paper is organized as follows. In Section 2, the problem is formulated, the CG method of optimization is described and ways to implement the algorithm efficiently in the adaptive filtering context are also discussed. In Section 3, the new block CG algorithm and its FD implementation are derived. Furthermore, different ways of accelerating convergence are presented. In Section 4 the adaptive channel equalization case is treated. Finally, simulation results are provided in Section 5 and the work is concluded in Section 6.