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Robust estimation in flat fading channels under bounded channel uncertainties

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ABSTRACT

We investigate channel equalization problem for time-varying flat fading channels under bounded channel uncertainties. We analyze three robust methods to estimate an unknown signal transmitted through a time-varying flat fading channel. These methods are based on minimizing certain mean-square error criteria that incorporate the channel uncertainties into their problem formulations instead of directly using the inaccurate channel information that is available. We present closed-form solutions to the channel equalization problems for each method and for both zero mean and nonzero mean signals. We illustrate the performances of the equalization methods through simulations.

1. Introduction

In this paper, we study channel equalization problem for time-varying flat (frequency-nonselective) fading channels under bounded channel uncertainties [1–7]. In this widely studied framework, an unknown desired signal is transmitted through a discrete-time time-varying channel and corrupted by additive noise where the mean and variance of the desired signal is assumed to be known. Although the underlying channel impulse response is not known exactly, an estimate and an uncertainty bound on it are given [4–6]. Here, we investigate three different channel equalization frameworks that are based on minimizing certain mean-square error criteria. These channel equalization frameworks incorporate the channel uncertainties into their problem formulations to provide robust solutions to the channel equalization problem instead of directly using the inaccurate channel information that is available to equalize the channel. Based on these frameworks, we analyze three robust methods to equalize time-varying flat fading channels. The first approach we investigate is the affine minimax equalization method [5,8,9], which minimizes the estimation error for the worst case channel perturbation. The second approach we study is the affine minimin equalization method [6,10], which minimizes the estimation error for the most favorable perturbation. The third approach is the affine minimax regret equalization method [4,5,11,7], which minimizes a certain "regret" as defined in Section 2 and further detailed in Section 3. We provide closed-form solutions to the affine minimax equalization, the minimin equalization and the minimax regret equalization problems for both zero mean and nonzero mean signals. Note that the nonzero mean signals frequently appear in iterative equalization applications [11,12] and equalization with these signals under channel uncertainties is particularly important and challenging.

When there are uncertainties in the channel coefficients, one of the prevalent approaches to find a robust solution to the equalization problem is the minimax equalization method [9,5,8]. In this approach, affine equalizer coefficients are chosen to minimize the MSE with respect to the worst possible channel in the uncertainty bounds. We emphasize that although the minimax equalization framework has been introduced in the context of statistical signal processing literature [9,5,8], our analysis significantly differs since we provide a closed-form solution to the minimax equalization problem for time-varying flat fading channels. In [5], the uncertainty is in the noise covariance matrix and the channel coefficients are assumed to be perfectly known. Furthermore, note that in [8], the minimax estimator is formulated as a solution to a semidefinite programming (SDP) problem, unlike in here. In this paper, the uncertainty is in the channel impulse response and we provide an explicit solution to the minimax channel equalization problem.

Although the minimax equalization method is able to minimize the estimation error for the worst case channel perturbation, however, it usually provides unsatisfactory results on the average [6]. An alternative approach to the channel equalization problem is the minimin equalization method [6,10]. In this approach, equalizer parameters are selected to minimize the MSE with respect to the most favorable channel over the set of allowed perturbations. Although the minimin approach has been studied in the literature [6,10], however, we emphasize that to the best of our knowledge, this is the first closed-form solution to the minimin channel equalization problem for time-varying flat fading channels.

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The minimin approach is highly optimistic, which could yield unsatisfactory results, when the difference between the underlying channel impulse response and the most favorable channel impulse response is relatively high [6]. In order to preserve robustness and counterbalance the conservative nature of the minimax approach, the minimax regret approaches have been introduced in the signal processing literature [4,13,7]. In this approach, a relative performance measure, i.e., "regret", is defined as the difference between the MSE of an affine equalizer and the MSE of the affine minimum MSE (MMSE) equalizer [7]. The minimax regret channel equalizer seeks an equalizer that minimizes this regret with respect to the worst possible channel in the uncertainty region. Although this approach has been investigated before, the minimax regret estimator is formulated as a solution to an SDP problem [4], unlike here. In this paper, we explicitly provide the equalizer coefficients and the estimate of the desired signal.

Our main contributions are as follows. We first formulate the affine equalization problem for time-varying flat fading channels under bounded channel uncertainties. We then investigate three robust approaches; affine minimax equalization, affine minimin equalization, and affine minimax regret equalization for both zero mean and nonzero mean signals. The equalizer coefficients, and hence, the MSE of each methods have been explicitly provided, unlike in [4,5,8,6,7].

The paper is organized as follows. In Section 2, the basic transmission system is described, along with the notation used in this paper. We present the affine equalization approaches in Section 3. First, we study the affine minimax equalization tuned to the worst possible channel filter. We then investigate the minimin approach and the minimax regret approach, and provide the explicit solutions to the corresponding optimization problems. In addition, we present and compare the MSE performances of all robust affine equalization methods in Section 4. Finally, we conclude the paper with certain remarks in Section 5.

2. System description

In this section, we provide the basic description of the system studied in this paper. Here, the signal x_t is transmitted through a discrete-time time-varying channel with a channel coefficient h_t, where x_t is unknown and random with known mean $\overline{x_t} \triangleq E[x_t]$ and variance $\sigma_x^2 \triangleq E[(x_t - \overline{x_t})^2]$. The received signal y_t is given by

$$y_t = x_t h_t + n_t, \tag{1}$$

where the observation noise n_t is independent and identically distributed (i.i.d.) with zero mean and variance σ_n^2 and independent from x_t. We consider a time-varying flat fading channel, where the bandwidth of the transmitted signal x_t is much smaller than the channel bandwidth so that the multipath channel simply scales the transmitted signal [14,15]. However, instead of the true channel coefficient, an estimate of h_t is provided as $\tilde{h}_t$, where $\delta h_t \triangleq \tilde{h}_t - h_t$ is the uncertainty in the channel coefficient and is modeled by $|h_t - \tilde{h}_t| = |\delta h_t| \leqslant \epsilon$, $\epsilon > 0$, $\epsilon < \infty$, where ϵ or a bound on ϵ is known.

We then use the received signal y_t to estimate the transmitted signal x_t as shown in Fig. 1. The estimate of the desired signal is given by

$$\begin{aligned}\hat{x}_t &= w_t y_t + l_t \\ &= w_t(x_t h_t + n_t) + l_t,\end{aligned} \tag{2}$$

where w_t is the equalizer coefficient. We note that in (2), the equalizer is "affine" where there is a bias term l_t since the transmitted signal x_t, and consequently the received signal y_t, are not necessarily zero mean and the mean sequence $\bar{y}_t \triangleq E[y_t]$ is not known due to uncertainty in the channel.

Even under the channel uncertainties, the equalizer coefficient w_t and the bias term l_t can be simply optimized to minimize the MSE for the channel that is tuned to the estimate $\tilde{h}_t$, which is also known as the MMSE estimator [16]. The corresponding equalizer coefficient and the bias term are given by [17,11]

$$\{w_{0,t}, l_{0,t}\} = \arg\min_{w,l} E\big[\big(x_t - w(\tilde{h}_t x_t + n_t) - l\big)^2\big]. \tag{3}$$

However, the estimate

$$\hat{x}_{0,t} \triangleq w_{0,t} y_t + l_{0,t}$$

may not perform well when the error in the estimate of the channel coefficient is relatively high [18,4,5]. One alternative approach to find a robust solution to this problem is to minimize a worst case MSE, which is known as the minimax criterion, as

$$\begin{aligned}&\{w_{1,t}, l_{1,t}\} \\ &\quad = \arg\min_{w,l} \max_{|\delta h_t| \leqslant \epsilon} E\big[\big(x_t - w\big((\tilde{h}_t + \delta h_t)x_t + n_t\big) - l\big)^2\big],\end{aligned} \tag{4}$$

where $w_{1,t}$ and $l_{1,t}$ minimize the worst case error in the uncertainty region [8,16]. However, this approach may yield highly conservative results, since the estimate

$$\hat{x}_{1,t} \triangleq w_{1,t} y_t + l_{1,t}$$

is formed by using the equalizer coefficient $w_{1,t}$ and the bias term $l_{1,t}$ that minimize the worst case error, i.e., the error under the worst possible channel coefficient [6,4,5]. Instead of this conservative approach, another useful method to estimate the desired signal is the minimin approach, where the equalizer coefficient and the bias term are given by

$$\begin{aligned}&\{w_{2,t}, l_{2,t}\} \\ &\quad = \arg\min_{w,l} \min_{|\delta h_t| \leqslant \epsilon} E\big[\big(x_t - w\big((\tilde{h}_t + \delta h_t)x_t + n_t\big) - l\big)^2\big],\end{aligned} \tag{5}$$

where $w_{2,t}$ and $l_{2,t}$ minimize the MSE in the most favorable case, i.e., the MSE under the best possible channel coefficient [6]. The estimate of the transmitted signal x_t is given by

$$\hat{x}_{2,t} \triangleq w_{2,t} y_t + l_{2,t}.$$

A major drawback of the minimin approach is that it is a highly optimistic technique, which could yield unsatisfactory results, when the difference between the actual and the best channel coefficients is relatively high [6].

In order to reduce the conservative characteristic of the minimax approach as well as to maintain robustness, the minimax regret approach is introduced, which provides a trade-off between

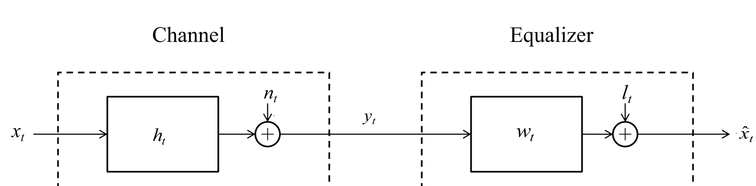

Fig. 1. A basic affine equalizer framework.