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A note on the improved LMI-based criterion for global asymptotic stability of 2-D state-space digital filters described by Roesser model using two's complement overflow arithmetic

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ABSTRACT

This paper focuses on some critical issues in a recently reported approach [V. Singh, Improved LMI-based criterion for global asymptotic stability of 2-D state-space digital filters described by Roesser model using two's complement overflow arithmetic, Digital Signal Process. 22 (2012) 471–475] for the global asymptotic stability of two-dimensional (2-D) fixed-point state-space digital filters described by the Roesser model employing two's complement overflow arithmetic. In particular, it is highlighted that the situation where Singh's approach can be applied to ensure the global asymptotic stability of digital filters in the presence of two's complement overflow nonlinearities is not conceivable.

1. Introduction

In the past few decades, a vast amount of research was devoted to the study of the system theoretic problems of two-dimensional (2-D) discrete dynamical systems. Such a study has predominantly been motivated by a wide variety of applications of 2-D discrete systems in many areas such as digital data filtering, image data processing and transmission, seismographic data processing, gas filtration, water stream heating, thermal processes in chemical reactors, 2-D digital control systems, river pollution modeling, grid based wireless sensor networks, modeling of partial differential equations, etc. [1–6].

In the implementation of stable and linear recursive digital filters using finite wordlength fixed-point processors, nonlinear effects are introduced owing to the overflow and quantization [7–9]. These nonlinear effects may result in the instability of the designed filter. Therefore, a major concern is to find the conditions under which the filter would be globally asymptotically stable. Many publications relating to the issue of stability of digital filters have appeared (see, for instance, [7–25] and the references cited therein). The common types of overflow nonlinearities are saturation, zeroing, triangular and two's complement [9,24]. The two's complement arithmetic adder is known to be the cheapest among all overflow arithmetic adders employed in practice. The effect of two's complement overflow nonlinearities on the stability of one-dimensional (1-D) digital filters has been investigated in [8,11–17]. The stability properties of 2-D digital filters have been studied in the presence of two's complement overflow [18–22], various combinations of quantization and overflow [23], generalized overflow [24], and quantization [25].

This paper is directly inspired by a recent paper [22]. In [22], Singh has reported a criterion for the global asymptotic stability of zero-input 2-D digital filters described by the Roesser model [26] implemented with two's complement arithmetic for the addition operation. Splitting the two's complement nonlinearity sector $[-1, 1]$ into two smaller sectors $[0, 1]$ and $[-1, 0]$ together with making use of a certain 'assumption' is a key step in the approach of [22]. The approach in [22] appears to be quite distinct from other approaches [8,11–21,24].

The purpose of this paper is to highlight that, for the systems implemented with two's complement overflow adders, the 'assumption' made in [22] is a major restriction in disguise and it is hard to conceive any situation where Singh's approach [22] can be applied to ensure the global asymptotic stability of digital filters in the presence of two's complement overflow nonlinearities. It will be demonstrated that all the examples considered in [22] fail to validate this 'assumption' and, consequently, the approach [22] is incapable of detecting the global asymptotic stability. Further, the implication of Singh's approach [22] is discussed in the context of 1-D digital filters with two's complement overflow arithmetic.

2. System description and criteria for global asymptotic stability

Consider the state-space quarter-plane nonlinear 2-D system described by the Roesser model [26]:

$$\boldsymbol{x}_{11}(k,l) = \begin{bmatrix} \boldsymbol{x}^h(k+1,l) \\ \boldsymbol{x}^v(k,l+1) \end{bmatrix} = \boldsymbol{f}\big(\boldsymbol{y}(k,l)\big) = \begin{bmatrix} \boldsymbol{f}^h(\boldsymbol{y}^h(k,l)) \\ \boldsymbol{f}^v(\boldsymbol{y}^v(k,l)) \end{bmatrix}, \quad (1a)$$

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$$\begin{aligned}\boldsymbol{y}(k,l) &= \left[\begin{array}{c}\boldsymbol{y}^h(k,l)\\ \hline \boldsymbol{y}^v(k,l)\end{array}\right]\\ &= \left[\begin{array}{cccc|cccc} y_1^h(k,l) & y_2^h(k,l) & \cdots & y_m^h(k,l) & y_1^v(k,l) & y_2^v(k,l) & \cdots & y_n^v(k,l)\end{array}\right]^T\\ &= \left[\begin{array}{c|c}\boldsymbol{A}_{11} & \boldsymbol{A}_{12}\\ \hline \boldsymbol{A}_{21} & \boldsymbol{A}_{22}\end{array}\right]\left[\begin{array}{c}\boldsymbol{x}^h(k,l)\\ \hline \boldsymbol{x}^v(k,l)\end{array}\right] = \boldsymbol{A}\boldsymbol{x}(k,l),\end{aligned} \tag{1b}$$

$$\boldsymbol{f}^h\big(\boldsymbol{y}^h(k,l)\big) = \left[\, f_1^h\big(y_1^h(k,l)\big)\;\; f_2^h\big(y_2^h(k,l)\big)\;\; \cdots\;\; f_m^h\big(y_m^h(k,l)\big)\,\right]^T, \tag{1c}$$

$$\boldsymbol{f}^v\big(\boldsymbol{y}^v(k,l)\big) = \left[\, f_1^v\big(y_1^v(k,l)\big)\;\; f_2^v\big(y_2^v(k,l)\big)\;\; \cdots\;\; f_n^v\big(y_n^v(k,l)\big)\,\right]^T, \tag{1d}$$

$$k \geqslant 0, \quad l \geqslant 0, \tag{1e}$$

where $k \in Z_+$, $l \in Z_+$, and Z_+ denotes the set of nonnegative integers. The two space coordinates k and l are horizontal coordinate and vertical coordinate, respectively. The state vectors $\boldsymbol{x}^h(k,l) \in \boldsymbol{R}^m$ and $\boldsymbol{x}^v(k,l) \in \boldsymbol{R}^n$ convey information horizontally and vertically, respectively. The matrices $\boldsymbol{A}_{11} \in \boldsymbol{R}^{m\times m}$, $\boldsymbol{A}_{12} \in \boldsymbol{R}^{m\times n}$, $\boldsymbol{A}_{21} \in \boldsymbol{R}^{n\times m}$, $\boldsymbol{A}_{22} \in \boldsymbol{R}^{n\times n}$ are the coefficient matrices, and the superscript T denotes the transpose. The overflow nonlinearities $f_i^h(y_i^h(k,l))$, $i = 1,2,\ldots,m$ and $f_i^v(y_i^v(k,l))$, $i = 1,2,\ldots,n$ are characterized by

$$\left.\begin{array}{ll} f_i^h\big(y_i^h(k,l)\big) = y_i^h(k,l), & \text{if } \big|y_i^h(k,l)\big| \leqslant 1\\ \big|f_i^h\big(y_i^h(k,l)\big)\big| \leqslant 1, & \text{if } \big|y_i^h(k,l)\big| > 1\end{array}\right\},\quad i = 1,2,\ldots,m, \tag{2a}$$

$$\left.\begin{array}{ll} f_i^v\big(y_i^v(k,l)\big) = y_i^v(k,l), & \text{if } \big|y_i^v(k,l)\big| \leqslant 1\\ \big|f_i^v\big(y_i^v(k,l)\big)\big| \leqslant 1, & \text{if } \big|y_i^v(k,l)\big| > 1\end{array}\right\},\quad i = 1,2,\ldots,n. \tag{2b}$$

We observe that two's complement overflow nonlinearities belong to (2). As in [22] (see [8,13–17,21,24] also), we assume that the effects of quantization are negligible. Such an assumption is based on a common approximation (i.e., 'decoupling approximation' [8,9]) where the effects of quantization and overflow are treated as mutually independent or decoupled. This 'decoupling approximation' can be justified if the signals are represented by a sufficiently large number of bits [8,9]. It is understood that system (1) has a finite set of initial conditions, i.e., there exist two positive integers K and L such that [18–21,23,24]

$$\boldsymbol{x}^v(k,0) = \boldsymbol{0}, \quad k \geqslant K; \qquad \boldsymbol{x}^h(0,l) = \boldsymbol{0}, \quad l \geqslant L, \tag{3}$$

where $\boldsymbol{0}$ denotes the null vector or null matrix of appropriate dimension. In this context, it may be pointed out that the initial conditions used in [22] are wrongly defined as $\boldsymbol{x}^v(k,l) = \boldsymbol{0}$, $k \geqslant K$; $\boldsymbol{x}^h(k,l) = \boldsymbol{0}$, $l \geqslant L$. Following [22], it can be verified that the initial conditions are required to be chosen as (3) in order to arrive at [22, Theorem 1].

Eqs. (1)–(3) represent a class of 2-D discrete dynamical systems implemented in finite wordlength fixed-point processors using two's complement signal representation. Examples of such systems are common in engineering and include 2-D digital filters with two's complement overflow arithmetic, 2-D nonlinear digital control systems, models of various physical phenomena (e.g., compartmental systems, single carriageway traffic flow [25], etc.), various dynamical processes represented by the Darboux equation [3] and so on. An example of the system represented by (1)–(3) can be found in wireless sensor networks [5], where each node typically employs 16-bit fixed-point microprocessors for data processing. Thus, the nonlinearities due to finite wordlength are inherently present in such systems.

Note that the nonlinear function while satisfying (2) belongs to the sector $[-1, 1]$. Next, consider the sector given by $[0, 1]$, i.e.,

$$f_i^h(0) = 0, \quad 0 \leqslant f_i^h\big(y_i^h(k,l)\big)y_i^h(k,l) \leqslant y_i^{h^2}(k,l),\quad i = 1,2,\ldots,m, \tag{4a}$$

$$f_i^v(0) = 0, \quad 0 \leqslant f_i^v\big(y_i^v(k,l)\big)y_i^v(k,l) \leqslant y_i^{v^2}(k,l),\quad i = 1,2,\ldots,n, \tag{4b}$$

and the sector given by $[-1, 0]$, i.e.,

$$f_i^h(0) = 0, \quad -y_i^{h^2}(k,l) \leqslant f_i^h\big(y_i^h(k,l)\big)y_i^h(k,l) \leqslant 0,\quad i = 1,2,\ldots,m, \tag{5a}$$

$$f_i^v(0) = 0, \quad -y_i^{v^2}(k,l) \leqslant f_i^v\big(y_i^v(k,l)\big)y_i^v(k,l) \leqslant 0,\quad i = 1,2,\ldots,n. \tag{5b}$$

Recently, Singh [22] has studied the problem of global asymptotic stability of the 2-D system (1)–(3). In analyzing the stability of the equilibrium $\boldsymbol{x}_e = \boldsymbol{0}$ of the 2-D system (1)–(3), [22] makes use of the following assumption.

Assumption 1. (See Singh [22].) All nonlinearities (2) in the system (1) at a time either belong to (4) or to (5), but not to both (4) and (5).

It may be mentioned that, in the case of 1-D systems, a temporal variable (time) plays a central role. Ref. [22] deals with a class of nonlinear 2-D discrete systems in which time does not have a role. Indeed, the independent variables of interest are spatial variables. Therefore, in the context of 2-D discrete systems, Assumption 1 may be modified as follows.

Assumption 2. For any set of initial conditions satisfying (3), all nonlinearities (2) in the system (1) along the line $k + l = p$ either belong to (4) or to (5) (but not to both (4) and (5)) for any nonnegative integer p.

Now, the main result of [22] may be stated as follows.

Theorem 1. *(See Singh [22].) Under Assumption 2, the equilibrium $\boldsymbol{x}_e = 0$ of the system (1)–(3) is globally asymptotically stable if there exist positive definite symmetric matrices $\boldsymbol{P}^h \in R^{m\times m}$, $\boldsymbol{P}^v \in R^{n\times n}$ and a positive definite diagonal matrix $\boldsymbol{D}$ such that*

$$\begin{bmatrix}\boldsymbol{P} & -\boldsymbol{A}^T\boldsymbol{D}\\ -\boldsymbol{D}\boldsymbol{A} & 2\boldsymbol{D} - \boldsymbol{P}\end{bmatrix} > \boldsymbol{0}, \tag{6}$$

where $\boldsymbol{P} = \boldsymbol{P}^h \oplus \boldsymbol{P}^v = \begin{bmatrix}\boldsymbol{P}^h & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{P}^v\end{bmatrix}$ and '$>$' signifies that the matrix is positive definite.

Remark 1. Observe that, in the approach of [22], condition (6) alone cannot ensure the global asymptotic stability of (1)–(3). If the system fails to confirm Assumption 2, the global asymptotic stability of the system cannot be established via the approach in [22]. In view of the characteristics of the two's complement overflow nonlinearities, it is not natural to expect that Assumption 2 is valid automatically for all kinds of systems given by (1)–(3). The state trajectories of the 2-D system (1)–(3) depend on the state matrix A for a given set of initial conditions. Thus, the validity of Assumption 2 is also dependent on the system parameters and initial conditions.

3. Critical issues associated with Assumption 2

Define

$$s_i = \sum_{j=1}^{m+n} |a_{ij}|, \quad i = 1,2,\ldots,(m+n), \tag{7}$$