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A novel two-stage nonrecursive architecture for the design of generalized comb filters

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ABSTRACT

This paper presents a novel two-stage class of decimation filters with superior spurious signal rejection performance around the so-called folding bands, i.e., frequency intervals whose signals get folded down to baseband due to decimation. The key idea to enhance signal rejection in the frequency domain lies on an effective way to place the zeros of a classical comb filter in the aforementioned folding bands. On the other hand, the paper provides a mathematical framework for designing two-stage multiplierless and nonrecursive structures of the proposed filters.
Examples are provided to highlight the key steps in the design of the proposed filters. Moreover, the frequency behavior of the proposed filters in both baseband and stopband is compared with classical and generalized comb filters, and a droop compensator is proposed to counteract the passband distortion of the proposed filters.

1. Introduction

Multirate architectures for sampling rate conversion have been under investigation for many years [1]. From a practical point of view, the decimation of a highly oversampled signal is accomplished using a cascade of two (or more) stages of decimation [1]. The first stage is usually a comb filter of order N decimating by a factor M, while the last stage employs an FIR filter providing the required selectivity on the sampled signal down-converted to baseband. Moreover, the design of a decimation stage in a multistage architecture imposes stringent constraints only on the shape of the filter frequency response over the so-called folding bands.

Let us briefly highlight the meaning of folding bands in connection to the first decimation stage in a multistage architecture. Consider an analog signal $x(t)$ with baseband bandwidth $[-B_x, +B_x]$ sampled by an A/D converter at the frequency $f_s = \rho 2B_x$ (ρ is the oversampling ratio; by definition, it is $\rho \geqslant 1$). The discrete-time sampled signal, $x[nT_s]$, presents normalized frequency bandwidth in the dimensionless frequency set $[-f_0, +f_0]$, with $f_0 = B_x/f_s = B_x/(\rho 2B_x) = 1/(2\rho)$. With this setup, the frequency response of the anti-aliasing filter employed in the first decimation stage should attenuate the quantization noise (QN) and any other undesired signal falling inside the frequency intervals defined as

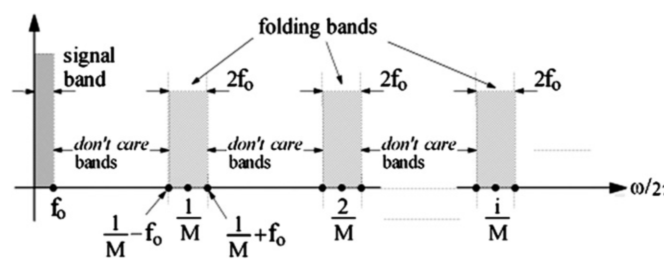

Fig. 1. Pictorial representation of the frequency intervals (folding bands) that should be carefully considered for the design of the first decimation stage.

$$\left[\frac{k}{M} - f_0, \frac{k}{M} + f_0\right], \quad \text{for } k = 1, \ldots, \lfloor M \rfloor. \tag{1}$$

The reason follows upon noting that the signals within these frequency bands will fold down to baseband because of the sampling rate reduction by M in the first decimation stage, thus irremediably affecting the signal resolution after the multistage decimation chain [1]. These frequency ranges are pictorially shown in Fig. 1.

Classical comb filters with z-transfer function [2]

$$H(z) = \frac{1}{M}\frac{1 - z^{-M}}{1 - z^{-1}} \tag{2}$$

have many advantages over other filters. Among them, the main advantage stems from the fact that their system function is extremely simple to implement and it does not require any multiplier. However, the magnitude response exhibits low attenuation

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in the folding bands, as well as a considerable passband droop that deteriorates the sampled signal.

With this background, let us provide a survey of the recent literature related to the problem addressed in this paper. Tutorials focusing on the design of multirate filters can be found in [1,3–7]. The design of optimized multistage decimation and interpolation filters has been recently addressed by Coffey in [8], while the design of multistage decimation architectures relying on constituent cyclotomic polynomial filters has been presented in [9,10]. Implementation aspects of comb filters with special emphasis on power consumption and overflow have been addressed in [11,12] where nonrecursive architectures relying on the polyphase decomposition were introduced. Methods [13,14] proposed compensation filters aimed to decrease the passband droop of comb filters. In [15], the authors proposed novel decimation schemes for $\sum\Delta$ A/D converters based on Kaiser and Hamming sharpened filters. These filters were improved in [16], then generalized in [17] for higher order decimation filters, and in [19] for wideband receivers. In [20], the authors introduced a two-stage architecture presenting better magnitude characteristics than the structure proposed in [16].

A 3rd-order modified decimation sinc filter was proposed in [21], and developed in [22]. The class of comb filters was generalized in [24], whereby the authors proposed an optimization framework for deriving the optimal zero rotations of Generalized Comb filters (GCFs) for any filter order and decimation factor. GCFs in [24] were proposed with the aim of increasing the $\sum\Delta$ QN rejection around the folding bands with respect to classical comb filters of equivalent order. In spite of the particular application to $\sum\Delta$ A/D converters, GCFs are pretty general in that they can be used in place of classical comb filters in multistage rate conversion architectures as the ones employed for signal extraction in broadband and reconfigurable digital receivers. Works [25,26] proposed compensation filters to decrease the passband droop of GC filters, while [27] addressed the design of efficient GC filters exploiting recursive architectures.

The considerations drawn above in connection to the frequency behavior of a classical comb filter around the folding bands, particularly around the first folding band, represent the very starting point toward the ideas proposed in this work. The main rationales are discussed in Section 2 where we also point out the main contributions of this work. The rest of the paper is organized as follows. In Section 3, we derive the z-transfer function of the proposed decimation filters. Section 4 presents a variety of design issues for improving the frequency response of the filters under investigation. Moreover, a mathematical framework for optimizing the zero location of the z-transfer function is developed. In Section 5, we present some design examples in order to highlight the choice of the parameters defining the behavior of the proposed filters. Finally, Section 6 draws the conclusion.

2. Motivation

One of the advantages of the decimation filters proposed in [24] stems from the optimization of the zeros locations of a classical Nth-order comb filter within the folding bands defined in (1). A nice consequence of this optimization consists in an increased attenuation in the folding bands. Given a decimation factor M, and neglecting the normalization constant ensuring unity gain at baseband, the z-transfer function of a 3rd-order GCF filter can be written as

$$H_{GCF3}(z) = \frac{1-z^{-M}}{1-z^{-1}} \frac{1-z^{-M}e^{j\alpha M}}{1-z^{-1}e^{j\alpha}} \frac{1-z^{-M}e^{-j\alpha M}}{1-z^{-1}e^{-j\alpha}}, \tag{3}$$

where α is the extent of the rotation undergone by the zeros of a classical comb filter. Using the notation summarized in the previous section about the folding bands, we note that a convenient

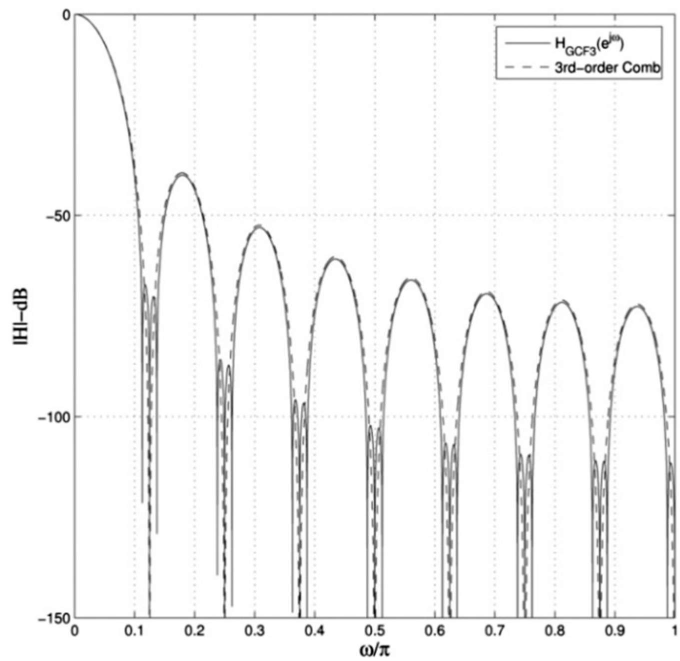

Fig. 2. Magnitude response (in dB) of the 3rd-order GCF filter in (3) compared with the magnitude response of a 3rd-order comb. Design parameters are as follows: $M = 16$, $\rho = 64$, and $\alpha = 0.03828$.

choice for α is $\alpha = q2\pi f_0$, with $q \in [-1, +1]$: this choice is such that the zeros of $H_{GCF3}(z)$ fall within the folding bands in (1). The magnitude of a 3rd-order GC filter is compared with the magnitude response of a classical 3rd-order comb filter in Fig. 2 for the following parameters: $M = 16$, $\rho = 64$, $\alpha = 0.03828$, and $q = 0.78$.

The drawback stemming from the zero-optimization is the introduction of multipliers in the first stage of decimation, as well as the impossibility to implement multiplierless recursive architectures due to the instability arising from imperfect zero-pole cancellation when coefficients quantization is exploited. In addition, GC filters exhibit passband droops of the same extent of classical comb filters.

The main contributions of this work stem from solving the aforementioned problems, while retaining the advantages of GC filters. We propose a novel class of decimation filters featuring the following advantages hereafter identified as conditions.

1. No need of a programmable FIR filter at low rate to compensate the passband droop introduced by comb filters.
2. Improved spurious signal rejection in the folding bands compared to traditional Nth-order comb filters (in this respect the proposed filters are superior as compared to comb filters).
3. Nonrecursive implementation not suffering from the instability problems arising from imperfect pole-zero cancellation due to finite precision effects.
4. Overall multiplierless filter design.

Let us briefly discuss in which way the present work differs from the previous work relevant to this one, developed by the same authors. Papers [9,10] addressed the design of multistage decimation filters based on cyclotomic polynomials. The problem was to solve an optimization problem for finding a proper combination of cyclotomic polynomials to meet a set of filter specifications. The design in [9,10] is different from the focus of this work since this very work proposes improved decimation filters stemming from classical comb filters. Paper [16] presented a sharpened comb decimator structure consisting of a cascade of a comb-filter based decimator and a sharpened comb decimator. In the attempt