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Sigma point Kalman filter for bearing only tracking

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Abstract

Relative merits of sigma point Kalman filters (SPKF), also known as unscented Kalman filters (UKF) vis-à-vis extended Kalman filter (EKF) and iterated extended Kalman filter (IEKF) for a bearing-only target-tracking problem using rms error and robustness with respect to outlier initial conditions are explored. After establishing that the rms error performance obtainable by SPKF/UKF and IEKF for this fairly severe non-linear system is similar to those obtainable from other competing techniques, the relative robustness of IEKF, SPKF/UKF and EKF with respect to large initial condition uncertainty (a common occurrence for this class of tracking problems) is investigated. Using several versions of SPKF/UKF, it is shown that SPKF is about 20 times more robust compared to EKF. It is illustrated that the additional design freedom available with a *scaled* version of SPKF/UKF may be utilised for further improvement of the robustness. The main contribution of this paper is quantification of relative robustness of these non-linear filters. A simplified criterion is suggested and used for quantifying track loss and the relative occurrence of such track loss in batch Monte Carlo simulation has been used as a measure of (lack of) robustness. As the SPKF/UKF does not introduce substantial computational burden, when compared to EKF, it is argued that SPKF/UKF algorithm may become a strong candidate for on-board implementation.

Keywords: Bearings-only tracking; Estimation algorithm; Extended Kalman Filter; Unscented Kalman Filter; Filtering techniques; Monte Carlo simulation; Target tracking filters; Sigma point filters

1. Introduction

Recent references indicate continued interest in bearing-only tracking (BOT) problem [1–7] with sonar and IR-based tracking systems. Civil applications include sonar-based robotic navigation and TV camera-based people tracking. BOT with a single tracker necessitates that the tracking platform must be moving. For tactical reasons, the tracker needs to be able to outmanoeuvre the target. Doğançay [4,5] has recently re-opened the issue of fixed target localisation with a moving observer whereas a constant acceleration moving target case has been discussed by the same author in Ref. [6], and by others [1,7]. Lin et al. [1] have presented a comparative study of several nonlinear, sub-optimal filters for a kinematically simple 2D BOT problem, with the target and the platform both moving with average constant velocities perturbed by random disturbances with known statistics. As a part of our effort to select suitable recursive filtering technique

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for onboard implementation in somewhat more complex three-dimensional tracking situations with poor observability problem, the kinematically simpler 2-D BOT problem considered in Ref. [1] was adopted to obtain more insights into filtering techniques. Compared to (the non-recursive MLE technique employed in) Ref. [6], the formulation in Ref. [1] treats the case where the observer and the target move in nearly parallel and constant (on the average) velocities.

The EKF, which had been a natural choice [6] for non-linear tracking problems, uses local slope linearisation [4] approach for calculating the mean and covariance of the random variables, which undergo a non-linear transformation. Such approximations are valid only for "small" perturbations around the mean. As EKF does not take cognisance of the amplitude of deviations during its computational cycle, errors may accumulate and the EKF loses track, as discussed in Refs. [1,6]. The incidence of track loss can demonstrably be reduced by a change of coordinate [6,7] and also by using pseudo-measurement approximation. In the present work we continue to use a Cartesian formulation, as we wanted to "stress" the filtering algorithm to obtain more insight into the techniques adopted. The performance of EKF for a strongly non-linear problem is often improved by iterating the EKF method such that the Jacobian at the assumed and actual operating points (e.g. prior and posterior estimates) become close enough [3,9]. We study the efficacy of this approach for the current tracking problem.

Recently, another sub-optimal method, called unscented Kalman filter (UKF) [10–14] also known as sigma point Kalman filter (SPKF) [15], is being tried out for non-linear problems where EKF gives unacceptable performance. The UKF/SPKF overcomes the flaws of EKF by utilising a deterministic "support points" approach to calculate mean and covariance terms, but cannot altogether avoid the problems of non-Gaussianity.

To explore the possibility of using several variants of SPKF for the 2D BOT problem, and for performance comparison with EKF, we use identical truth model with Lin et al. [1]. For a robustness comparison, we count the relative occurrence of track loss cases in EKF, IEKF and SPKF by using large batch runs of Monte Carlo simulation, the motivation for which is given in Ref. [7].

Earlier workers [1,3] have not defined the term 'track-loss'. For an objective analysis of track loss, we have proposed a simple numerical criterion based on the position error bound (X_{limit}) at the end of the filter run.

For this work we use the scaled SPKF form [13], which provides additional free parameters for tuning the filter. We study the effect of one such parameter α, which controls the spread of the sigma points, on filtering and robustness performances.

A common form of accommodating initial condition uncertainty is to use larger values of initial error covariance $P(0)$. We investigate whether such heuristics are applicable for the present problem as well. Zarchan and Musoff [8] has demonstrated the disadvantage of using large value of $P(0)$ for a non-linear projectile tracking problem. Our approach is somewhat different as our motivation is to determine whether by enhancing the value of $P(0)$ in the filter compared to the 'truth' value it is possible to capture more outliers and thereby reduce the occurrence of track loss. Zarchan and Musoff [8] has changed the initial covariance simultaneously for truth model and filter.

2. Problem formulation

The non-linear BOT problem used in Ref. [1] has two components, namely, the target kinematics, and the tracking *platform* kinematics as shown in Fig. 1. The tracking platform moves approximately parallel to the target with approximately constant velocity, described by the following discrete time equations:

$$x_p(k) = \bar{x}_p(k) + \Delta x_p(k), \quad k = 0, 1, \ldots, n_{step}, \tag{1}$$

$$y_p(k) = \bar{y}_p(k) + \Delta y_p(k), \quad k = 0, 1, \ldots, n_{step}, \tag{2}$$

where $\bar{x}_p(k)$ and $\bar{y}_p(k)$ are the (known) average platform position co-ordinates and $\Delta x_p(k)$ and

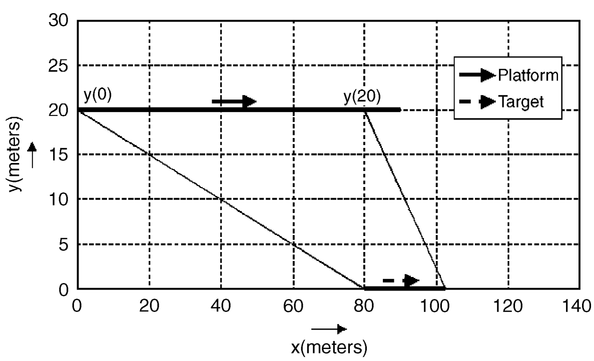

Fig. 1. Tracking Kinematics.