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Design of variable and adaptive fractional order FIR differentiators

Chien-Cheng Tseng

[a] *Department of Computer and Communication Engineering, National Kaohsiung First University of Science and Technology, Kaohsiung, Taiwan*

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Abstract

In this paper, the design problems of variable and adaptive fractional order finite impulse response (FIR) differentiators are investigated. First, the fractional differencing method and weighted least squares (WLS) approach are presented to design variable fractional order differentiators which can be implemented by the efficient Farrow structure. Next, an adaptive fractional order differentiator is developed and applied to estimate the parameters of $1/f$ noise from the finite observation data set. The parameters are updated by using least mean squares (LMS) adaptive algorithm. Finally, the variable fractional order differentiator is used to reduce the error rate of handwritten signature verification system.
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1. Introduction

During the past three decades, the fractional calculus has received great attentions in many engineering applications and science including fluid flow, automatic control, electrical networks, electromagnetic theory and image processing [1–6]. The integer order n of derivative $D^n f(x) = \mathrm{d}^n f(x)/\mathrm{d}x^n$ of function $f(x)$ is generalized to fractional order $D^{\upsilon} f(x)$, where υ is a real number. One of the important research topics in fractional calculus is to implement the fractional operator D^{υ} in continuous and discrete time domains. An excellent survey of this implementation has been presented in the recent paper [7]. For continuous time case, some methods for obtaining an approximated rational function using evaluation, interpolation and curve fitting techniques have been studied. These methods include Carlson's method, Roy's method, Chareff's method and Oustaloup's method [8–11]. For discrete time case, there have been several methods presented to design FIR and IIR filters for implementing operator D^{υ}, including fractional differencing formula or Euler method, trapezoidal rule or Tustin method, continued fraction expansion, optimization method and Prony's method [12–17].

On the other hand, the variable digital filter design has become an important research topic in recent years [18–27]. The main feature of variable filter is that the frequency characteristics can be quickly changed without re-designing a new filter,

E-mail address: tcc@ccms.nkfust.edu.tw.

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so the variable filter is particularly useful for on-line signal processing. Generally speaking, the design of variable digital filter can be classified into following two categories. One is the digital filters with adjustable magnitude response. This kind of filter is useful in audio signal processing and noise reduction [19,20]. The other is the digital filter with adjustable fractional delay response. This type of filter has been widely used in the applications of time adjustment in digital receiver, antenna array processing, speech coding and synthesis, modeling of music instruments and A/D conversion, etc. [21–26]. An excellent survey of the variable fractional delay filter design has been presented in tutorial paper [21]. Except the above two kinds of filters, the design of digital filters with adjustable magnitude and fractional delay responses have been also considered recently [27].

In this paper, we will study the discrete-time implementation of fractional differential operator D^{υ}, that is, the design of fractional order digital differentiator (FODD). In the conventional FODD designs, the order υ is assumed to be a fixed fractional number. In this paper, we will extend the order υ to be an adjustable fractional number. Thus, it is a research branch of variable filter design. In [28], the Riemann–Liouville definition of fractional operator with variable order has been introduced and the behavior has been also studied. In Section 2, the fractional differencing method and weighted least squares (WLS) approach are presented to design variable fractional order differentiators which can be implemented by the efficient Farrow structure. In Section 3, an adaptive fractional order differentiator is developed and applied to estimate the parameters of $1/f$ noise from the finite observation data set. The parameters are updated by using least mean squares (LMS) adaptive algorithm. In Section 4, the fractional order differentiator is used to reduce the error rate of handwritten signature verification system. Finally, a conclusion is made.

2. Variable fractional order differentiator

In this section, two methods will be presented to design variable fractional order FIR differentiator whose ideal frequency response is given by

$$D(\omega,\upsilon) = (j\omega)^{\upsilon} e^{-jI\omega}, \tag{1}$$

where I is a prescribed delay and υ is a variable or adjustable fractional number in the range $[-0.5,0.5]$. Because practical digital filters will introduce time delay, the linear phase term $e^{-jI\omega}$ is considered in the ideal response, see Eq. (24) in [15]. Clearly, the magnitude response of $D(\omega,\upsilon)$ is equal to ω^{υ} and the phase response is $-I\omega + 0.5\upsilon\pi$. The transfer function of the variable FIR filter used to approximate this specification is chosen as follows:

$$H(z,\upsilon) = \sum_{n=0}^{N} h_n(\upsilon) z^{-n}, \tag{2}$$

where $h_n(\upsilon)$ are the polynomial functions in υ of degree M, i.e.,

$$h_n(\upsilon) = \sum_{m=0}^{M} a_{nm} \upsilon^m. \tag{3}$$

Since $h_n(\upsilon)$ is real valued, the frequency response $H(e^{j\omega},\upsilon)$ is conjugate symmetric, i.e.,

$$H(e^{-j\omega},\upsilon) = H(e^{j\omega},\upsilon)^*, \tag{4}$$

where * denotes the complex conjugate. Substituting (3) into (2), the transfer function can be rewritten as

$$H(z,\upsilon) = \sum_{m=0}^{M}\sum_{n=0}^{N} a_{nm} z^{-n} \upsilon^m = \sum_{m=0}^{M} G_m(z)\upsilon^m, \tag{5}$$

where $G_m(z) = \sum_{n=0}^{N} a_{nm} z^{-n}$. Thus, the filter $H(z,\upsilon)$ can be implemented by the efficient Farrow structure shown in Fig. 1 once the sub-filters $G_m(z)$ have been designed. Because the sub-filters $G_m(z)$ are all fixed, we can adjust the parameter υ to change the order of the differentiator. Now, the design problem becomes how to find a_{nm} such that the frequency response $H(e^{j\omega},\upsilon)$ fits $D(\omega,\upsilon)$ as well as possible. In the following, two methods are developed to solve this design problem. One is the fractional differencing method, the other is the WLS method. The details are described below.

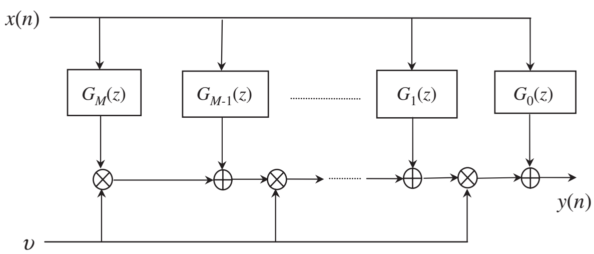

Fig. 1. The Farrow structure for variable fractional order differentiator, where order υ is adjustable.

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