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Approximation and identification of diffusive interfaces by fractional models

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Abstract

Heat transfer problems obey to diffusion phenomenon. In this paper we show that they can be modelled with the help of fractional systems. The simulation is based on a fractional integrator operator where the non-integer behaviour acts only over a limited spectral band. Starting with frequency considerations derived from the analysis of a diffusion problem, a more general approximation of the fractional system is proposed. A state-space model is presented that gives an accurate simulation for transients, and with which it is possible to carry out an output-error technique to estimate the model parameters. Numerical simulations of the heat transfer problem are used to illustrate the improvements of the proposed model.

Keywords: Fractional systems; Non-integer systems; Fractional operator; Modelling; Estimation; Output-error identification; Heat transfer; Diffusive interfaces

1. Introduction

The diffusive interface is characterized by fractional order dynamic behaviour, i.e. characterized by long memory transients and infinite dimensional structure [1]. Physically, such phenomenon appear when the transients of the systems are governed by the diffusion equation. A well known example is the case of heat transfer where the flux and the temperature at the interface are interrelated through fractional order operators [2,3]. These dynamics also appear in the case of an induction machine, with Foucault currents inside rotor bars [4–7]. Other examples are found in electrochemistry [8] and viscoelasticity [9]. Solutions to such problems have been developed to model this phenomenon [8,10,11]. The major limitation of these approaches, including the one that we proposed (with one fractional integrator), is that they rely on the properties of the fractional order model, but do not take into account the characteristics of the diffusive phenomenon, particularly at the interface.

The objective of this article is to analyse the frequency behaviour of a diffusive interface, and to show how the problem geometry influences on the phase plot. Using this analysis, it is then possible to define the frequency objectives of the approximate

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fractional model and to propose a structure allowing to these objectives to be satisfied. Practically, this analysis shows that a model with one fractional integrator [12–14] cannot satisfy all the objectives, whereas a model with two fractional integrators, one of them being constrained to $n = 0.5$, gives a good fit to the simulated experiment. The numerical simulations presented in this paper show that this new fractional model permits to satisfy the most important frequency objectives, using system identification in the time domain.

1.1. Approximate modelling of a diffusive interface

Let us consider the classical "wall" problem used to analyse heat transfer [2], represented in Fig. 1.

Temperature $T(x, t)$ is assumed to be uniform on any plane parallel to the faces A and B. Let $\phi(x, t)$ be the heat flux passing through the wall at abscissa x. $T(x, t)$ and $\phi(x, t)$ are governed by heat diffusion equations (1) and (2).

$$\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T(x,t)}{\partial x^2}, \tag{1}$$

$$\phi(x,t) = -\lambda \frac{\partial T(x,t)}{\partial x} \tag{2}$$

with

- $\alpha = \lambda/\rho c$: diffusivity,
- λ: thermal conductivity,
- ρ: mass density,
- c: specific heat.

1.2. Diffusive interface

Eqs. (1) and (2) specify the relation between $\phi(x, t)$ and $T(x, t)$, respectively, considered as system input ($u(t)$) and output ($y(t)$) when $x = 0$, which define the diffusive interface; then $y(t) = T(0, t)$. The boundary conditions on the faces A and B are:

$$\begin{cases} \phi(0,t) = u(t), \\ \phi(L,t) = \dfrac{T(L,t)}{R}, \end{cases} \tag{3}$$

where R is the thermal resistance between the wall and the air (on face B). Because the model is carried out around an operating point, air temperature is assumed to be constant and equal to zero.

Thus, the modelling of this interface (at $x = 0$) is equivalent of the determination of the transfer function $H(s)$ between $Y(s)$ and $U(s)$ (where $Y(s)$ and $U(s)$ are, respectively, the Laplace transforms of $y(t)$ and $u(t)$):

$$H(s) = \frac{\lambda R\sqrt{(s/\alpha)} + 1 + \left(\lambda R\sqrt{(s/\alpha)} - 1\right)\mathrm{e}^{-(s/\alpha)L^2}}{\lambda\sqrt{s/\alpha}\left(\lambda R\sqrt{(s/\alpha)} + 1 - \left(\lambda R\sqrt{(s/\alpha)} - 1\right)\mathrm{e}^{-(s/\alpha)L^2}\right)}. \tag{4}$$

Let us consider that heat flux $\phi(0, t)$ is a step input whose value is ϕ. Then

$$T(0,s) = H(s)\,\frac{\phi}{s}. \tag{5}$$

If we consider $t \to \infty$ (or equivalently $s \to 0$) we get

$$T(0,\infty) = y(\infty) = R\phi \tag{6}$$

that is to say that the wall behaves like a thermal resistance equal to zero.

Reciprocally, at very short times ($t \to 0$ or $s \to \infty$) we get

$$H(s) \simeq \frac{\sqrt{\alpha}}{\lambda s^{0.5}} \tag{7}$$

that is to say that the wall behaves like a non-integer integrator whose order is equal to 0.5.

Remark. This phenomenon is not restricted to the heat diffusion, it is also observed in the case of induced currents in the rotor bars of an induction machine. A numerical simulation using finite elements [6] has permitted an estimate of the frequency response of this phenomenon for a trapesoid rotor bar (see Fig. 2). One can verify that for $\omega \to \infty$, order n tends to 0.5, characterizing diffusion phenomena. On the other hand, the influence of the bar geometry appears at intermediary frequencies: in this example, the phase exceeds $-45°$, that is to say that n is higher than 0.5 in the concerned frequency domain.

A first conclusion is that diffusive interfaces can be modelled using a fractional operator (or non-integer one) where the order $n = 0.5$ is

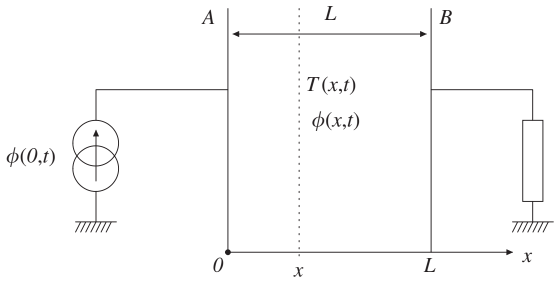

Fig. 1. Wall problem for heat transfer.