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A new nonstationary LMS algorithm for tracking Markovian time varying systems

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Abstract

We propose in this paper a new adaptive algorithm, designed to track system impulse responses, characterized by stochastic Markovian time variations. The proposed nonstationary least mean square (NSLMS) algorithm is designed so that it explicitly takes into account the structure of the nonstationarity. Hence, unlike the classical LMS algorithm, the NSLMS algorithm is not blind with respect to the time variations of the system impulse response to identify.

The proposed algorithm structure is based on a coupling between the estimation of the Markovian parameter that characterizes the nonstationarity and the estimation of the adaptive filter that identifies the system impulse response. The adaptive identification of the Markovian parameter is performed by an LMS algorithm, based on the minimization of the mean square of the system identification error.

A theoretical analysis of the transient and the steady-state behaviors of the NSLMS adaptive filter is carried out. In particular, an analytical expression of the step size that guarantees the stability of the latter is established. The theoretical misadjustment that measures the tracking ability of the NSLMS algorithm is computed for an i.i.d. input.

We prove that in the steady-state, the NSLMS algorithm exhibits better performance than the classical LMS algorithm, and goes beyond the limitation of the LMS algorithm to track severe filter time variations.

The experimental results reported here are in perfect agreement with the theory. They display the good properties of the NSLMS algorithm and demonstrate its ability to yield good performances in a hard Markovian time varying environment.

Keywords: Adaptive identification; Nonstationary; Markov model; LMS; NSLMS; Tracking ability

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1. Introduction

The least mean square (LMS) algorithm is the most widely used among gradient-based adaptation algorithms. It is known for its simplicity and its good steady-state performance in stationary context. The performance of the LMS algorithm in a nonstationary context, where the impulse response of the identified system varies in time, has been extensively analyzed in the literature. Many results on its steady-state misadjustment and its tracking performance have been established for the random walk model [1,2], or the first-order Markov model [3]. In this paper, we consider the case of first-order Markovian variations encountered in many realistic applications such as, transmission systems [4–7], acoustic echo cancellation [8], and audio signal processing [9]. In most studies, the assumption of slow system variations is made. However, when this assumption is relaxed, it is shown that the LMS algorithm cannot track some types of fast Markovian time variations. In these situations, it is better to switch off the adaptation once the transient period is completed [10–12]. In fact, the LMS algorithm is designed to estimate recursively the value of a fixed unknown filter. Consequently, the LMS adaptive identification is blind with respect to the real filter time variations and that is why the algorithm is not efficient in the case of severe time variations.

In order to guarantee better results than those obtained by the classical LMS algorithm, we propose in this paper, a new algorithm that can identify the Markovian time variation of the actual system. Indeed, it is designed to take into account prior knowledge on the nonstationarity's model of the filter to identify. The nonstationary LMS (NSLMS) algorithm constitutes a new approach that is different from those based on variable convergence factor [13–15]. These algorithms are based on the adaptation of the convergence factor with respect to the variations of the filter. Though they improve the tracking ability of the LMS algorithm, they are still blind with respect to the nonstationarity of the system. Moreover, the NSLMS algorithm does not require prior knowledge of the unknown statistics of the observation noise and the filter noise as it is the case in Kalman algorithms [16].

The remainder of the paper is organized as follows. In Section 2, we present the studied problem. The description of the design of the NSLMS algorithm is given in Section 3. In Section 4, the transient behavior of the NSLMS algorithm is studied. The mean convergence of the NSLMS adaptive filter is established from a theoretical viewpoint. The analytical expression of the NSLMS misadjustment is computed in Section 5. A theoretical study of the tracking ability of the NSLMS algorithm is carried out and its superiority over the LMS algorithm is demonstrated. The experimental results reported in Section 6 show a perfect agreement with theory and point out the attractive properties of the proposed NSLMS algorithm.

2. Presentation of the problem

In this paper, we are interested in the adaptive identification of Markovian time varying filters. The classical formulation of such filtering problem is depicted in Fig. 1. The noisy input/output equation of the filter is

$$y_k = F_k^{\mathrm{T}} X_k + n_k, \tag{1}$$

where $X_k = (x_k, x_{k-1}, \ldots, x_{k-N+1})^{\mathrm{T}}$ is the input vector. The input x_k is assumed to be a stationary random observable sequence, and the observation noise n_k is assumed to be statistically independent with zero mean value and power $P_n = E(n_k^2)$. The filter parameter vector $F_k = (f_0(k), f_1(k), \ldots, f_{N-1}(k))^{\mathrm{T}}$ is supposed to vary in time according to a first-order Markov model given by

$$F_k = aF_{k-1} + \Omega_k, \tag{2}$$

where $\Omega_k = (\omega_0(k), \ldots, \omega_{N-1}(k))^{\mathrm{T}}$ is the nonstationarity noise vector. Each noise $\omega_i(k)|_{i=0,\ldots,N-1}$, is an unknown white noise independent of x_k and n_k. We assume here that the noises $\omega_i(k)|_{i=0,\ldots,N-1}$ are