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Robust filtering for jumping systems with mode-dependent delays

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Abstract

In this paper, the filtering problem for a class of linear uncertain systems with Markovian jump parameters and functional time delay is examined. The uncertainties are time-varying and norm-bounded parametric uncertainties and the delay factor depends on the mode of operation. We provide complete results for robust weak-dependent stochastic stability and robust linear filter design. Then we extend the theoretical development to the case when a prescribed performance measure is desired. All the results are cast into convenient linear matrix inequality (LMI) forms.

Keywords: Markovian jump systems; Functional time delay; Stochastic stability; Robust filtering; $\mathscr{H}_\infty$ filtering; Uncertain parameters

1. Introduction

Filtering is perhaps one of the oldest problems studied in systems theory [1]. In recent years, robust filtering arose out of the desire to determine estimates of immeasurable state variables for dynamical systems with uncertain parameters. The past decade has witnessed major developments in robust filtering problem using various approaches [2–6]. In connection with system measurements and/or information flow amongst different parts of dynamical systems, time delay arises quite naturally [7] in modelling of industrial and engineering systems. Thus, the class of dynamical systems with time delay has attracted the attention of numerous investigators in the last two decades. Design of robust state estimators and observers to different

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classes of continuous-time systems with parametric uncertainties and state delay have been pursued in [8,9]. Markovian jump systems emerge when the physical models under consideration are subject to random changes [10–19]. This paper contributes to the further development of robust filtering of a class of uncertain jump time-delay systems and establish new results for the case in which the delay factor depends on the mode of operation and the uncertainties are time-varying and norm-bounded parametric uncertainties. We introduce the notion of stochastic stability with weak delay dependence and develop appropriate sufficient conditions. Then we design a linear Markovian filter which ensures that the augmented filtering system is mean-square quadratically stable for all admissible uncertainties. The results are then extended to $\mathscr{H}_\infty$-filtering. All the solvability conditions for the above problems are cast in the form of linear matrix inequalities (LMIs) [20,21].

Notations and facts: Let W^t, W^{-1} and $\lambda(W)$ to be the transpose, the inverse and the eigenvalues of the square matrix W; $W>0$ ($W<0$) denotes a positive- (negative-) definite matrix W. The open left-half of the complex plane is represented by $\mathbb{C}^-$. For a real symmetric matrix W, $W>0$ ($W<0$) stands for positive- (negative-) definite and means that $\lambda(W)$ are positive (negative). I stands for the identity matrix with appropriate dimension and $W^\dagger$ denote the pseudo-inverse of W. Let $\mathbb{C}_{n,\tau} = \mathbb{C}([-\tau, 0], \mathbb{R}^n)$ denote the Banach space of continuous vector functions mapping the interval $[-\tau, 0]$ into $\mathbb{R}^n$ with the topology of uniform convergence. If $\alpha \in \mathbb{R}$, $d \geqslant 0$ and $x \in \mathbb{C}([\alpha - \tau, \alpha + d], \mathbb{R}^n)$ then for any $t \in [\alpha, \alpha + d]$, we let $x_t \in \mathbb{C}$ be defined by $x_t(\theta) := x(t+\theta)$, $-\tau \leqslant \theta \leqslant 0$. If $\mathbb{D} \subset \mathbb{R} \times \mathbb{C}$, $f : \mathbb{D} \to \mathbb{R}^n$ is a given function, the relation $\dot{x}(t) = f(t, x_t)$ is a retarded functional differential equation (RFDE) on $\mathbb{D}$ where $x_t(t), t \geqslant t_o$, denotes the restriction of $x(\cdot)$ to the interval $[t-\tau, t]$ translated to $[-\tau, 0]$. Here, $\tau > 0$ is termed the *delay factor*. In the sequel, we let $\mathbb{E}$ stands for mathematical expectation. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

2. Functional time-delay jumping systems

2.1. Problem formulation

Given a probability space $(\Omega, \mathscr{G}, \mathbf{P})$ where Ω is the sample space, $\mathscr{G}$ is the algebra of events and $\mathbf{P}$ is the probability measure defined on $\mathscr{G}$. Let the random form process $\{\eta_t, t \in [0, \mathscr{T}]\}$ be a homogeneous, finite-state Markovian process with right continuous trajectories and taking values in a finite set $\mathscr{S} = \{1, 2, \ldots, s\}$ with transition probability from mode i at time t to mode j at time $t+\delta$, $i,j \in \mathscr{S}$:

$$p_{ij} = \Pr(\eta_{t+\delta} = j | \eta_t = i) = \begin{cases} \alpha_{ij}\delta + o(\delta) & \text{if } i \neq j, \\ 1 + \alpha_{ij}\delta + o(\delta) & \text{if } i = j \end{cases} \tag{2.1}$$

with transition probability rates $\alpha_{ij} \geqslant 0$ for $i,j \in \mathscr{S}$, $i \neq j$ and

$$\alpha_{ii} = -\sum_{m=1, m\neq i}^{s} \alpha_{im}, \quad \hat{\alpha} \triangleq \max_i \{|\alpha_{ii}|, i \in \mathscr{S}\}, \tag{2.2}$$

where $\delta > 0$ and $\lim_{\delta \downarrow 0} o(\delta)/\delta = 0$. The set $\mathscr{S}$ comprises various operational modes.

We consider a class of state-delay dynamical systems with Markovian jump parameters described over the space $(\Omega, \mathscr{G}, \mathbf{P})$ by

$$\begin{aligned} (\Sigma_J): \quad \dot{x}(t) &= [A_o(\eta_t) + \Delta A_o(t, \eta_t)]x(t) + [A_d(\eta_t) + \Delta A_d(t, \eta_t)]x(t - \tau_{\eta_t}) + [B_o(\eta_t) + \Delta B_o(t, \eta_t)]w(t) \\ &= A_{\Delta o}(t, \eta_t)x(t) + A_{\Delta d}(t, \eta_t)x(t - \tau_{\eta_t}) + B_{\Delta o}(t, \eta_t)w(t), \end{aligned} \tag{2.3}$$

$$x(t) = \phi(t) \in \mathscr{L}_2(-\max\{\tau_{\eta_t}\}, 0; \mathbb{R}^n), \qquad \eta_0 = i \in \mathscr{S}, \quad t \geqslant 0, \tag{2.4}$$