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Cyclostationary analysis with logarithmic variance stabilisation

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ABSTRACT

Second order cyclostationary (CS2) components in vibration or acoustic emission signals are typical symptoms of a wide variety of faults in rotating and alternating mechanical systems. The square envelope spectrum (SES), obtained via Hilbert transform of the original signal, is at the basis of the most common indicators used for detection of CS2 components. It has been shown that the SES is equivalent to an autocorrelation of the signal's discrete Fourier transform, and that CS2 components are a cause of high correlations in the frequency domain of the signal, thus resulting in peaks in the SES. Statistical tests have been proposed to determine if peaks in the SES are likely to belong to a normal variability in the signal or if they are proper symptoms of CS2 components. Despite the need for automated fault recognition and the theoretical soundness of these tests, this approach to machine diagnostics has been mostly neglected in industrial applications. In fact, in a series of experimental applications, even with proper pre-whitening steps, it has been found that healthy machines might produce high spectral correlations and therefore result in a highly biased SES distribution which might cause a series of false positives. In this paper a new envelope spectrum is defined, with the theoretical intent of rendering the hypothesis test variance-free. This newly proposed indicator will prove unbiased in case of multiple CS2 sources of spectral correlation, thus reducing the risk of false alarms.

1. Introduction

Significant second order cyclostationary (CS2) fault-symptomatic components have been found in a variety of mechanical systems: from IC engines, gears and bearings [1] to hydraulic turbines [2] and rotor–stator rub [3]. Second order cyclostationary signals are random signals characterised by a short time power spectral density (PSD) which varies periodically in time. The most common cyclostationary components in machine diagnostics are characterised by a constant PSD shape, which is simply amplitude modulated by a periodic signal. More rigorously, these signals show a periodic variation of their second order statistics, i.e. their autocovariance function (or instantaneous autocorrelation).

One of the most successful cyclostationary indicators is the squared envelope spectrum (SES). Its application to bearing diagnostics is prior to the development of the cyclostationary framework, but its direct second order cyclostationary nature has been demonstrated by Randall et al. [4]. More recently the substantial equivalence between narrow-band SES and the most common cyclic-spectral analysis indicators (cyclic power spectrum, cyclic coherence and cyclic modulation spectrum)

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has been proved and the use of envelope analysis as a proxy for second order cyclostationary detection has been shown as the most computational efficient approach among the ones proposed in literature [5,6].

The application of envelope analysis to CS2 signals can be described as follows:

- an initial band-pass filtering operation defines the area of the PSD of the signal which is suspected to carry CS2 components (the pass-band is generally identified by its central frequency f, called "spectral frequency");
- then an analytic signal transformation and a squaring of the absolute value of the analytic signal produce the squared-envelope of the band-pass filtered signal, representing the time domain energy fluctuation in the pass-band;
- finally a traditional discrete Fourier transform (DFT) of the squared-envelope allows obtaining the SES (in its complex form and/or in terms of magnitude with an additional absolute-square operation). The final step defines a second frequency domain (called "cyclic frequency" domain, symbol α), which describes the periodicity in the energy fluctuation.

If repeated for a series of band pass filters with different central frequencies, bi-spectral cyclostationary indicators are obtained, such as the cyclic modulation spectrum (CMS) [5], the envelope-based cyclic periodogram [6] and, through a normalisation step, the cyclic coherence.

Despite the recognition of the critical role of the SES and a wide literature on its application, few studies have specifically dealt with the establishment of rigorous statistical tests for the presence of CS2 components in the SES.

The first significant body of literature on the subject, specifically aimed at mechanical applications, is due to Antoni [7]. In this work it was highlighted how, if the signal is stationary, there would be no power fluctuation and consequently no periodicity in the squared-envelope. In this case, the complex SES (DFT of the squared-envelope) tends to a complex-normal distribution. Under the additional assumption of white noise (zero-mean delta-correlated process) the square-magnitude of the SES is distributed as a chi-square with two degrees of freedom and a variance depending only on the variance of the time domain signal and the parameters of the filter. Given the white noise assumption, a single scale factor is sufficient for this chi-square test. An extension of this approach to coloured noise signals has been proposed relaxing the white noise hypothesis and introducing a frequency dependent variance scale factor to take into account an uneven distribution of the signal's power along the spectral frequency axis [8].

Both techniques rely on the following assumptions:

- The variance is assumed to be known *a priori*, whereas it is actually estimated on the basis of the signal RMS, in the first case, and the autocorrelation of the signal's PSD in the second.
- The effect of a non-damage related CS2 component is limited to a series of harmonics of its characteristic cyclic frequency and does not influence the rest of the envelope spectrum.

The first point is mostly an issue of mathematical rigour, at least in case of long white-noise signals; however it is symptomatic of the weak spot of the test, related to the scale factor of the chi-square distribution at the different frequencies.

The second assumption has already been shown incorrect [8], because any CS2 component has a biasing effect on the whole cyclic axis.

This study proposes a new enveloping procedure and envelope spectrum (the log-envelope) which allows for a variance-free hypothesis test for cyclostationary components. The newly proposed indicator is not only an improvement in terms of mathematical rigour, but will also prove completely unbiased by multiple CS2 components.

This paper starts with a brief review of the traditional Hilbert-based enveloping procedure, followed by the introduction of the new log-envelope spectrum and the corresponding hypothesis test, inspired by the Priestly–Subba Rao (PSR) test for non-stationarity. This will be done first on the common envelope spectrum, obtained in time domain, and later extended to the double-frequency cyclic modulation spectrum, following the equivalence between cyclic power spectrum and narrow-band enveloping [5,6]. Then the paper will show the improved robustness of the statistical tests based on log-envelope indicators in comparison with traditional Hilbert-based quantities using both analytical–numerical studies and experimental examples. The latter consist of two series of bearing signals: one taken from the IMS bearing dataset, the other obtained on a newly developed test-rig for the simulation of wind turbine kinematics (Queensland University of Technology).

2. Full-spectrum time-domain enveloping

In this section two enveloping procedures will be discussed: the traditional Hilbert envelope (usually called *squared envelope*) and the log-envelope (newly proposed in this paper). The quantities will be defined in time and frequency domain (envelope and envelope spectrum) and their statistical properties will be analysed for the cases of stationary δ-correlated signals and pure 2nd order cyclostationary signals. Finally, drawing from these analytical results, the superior qualities of the newly proposed logarithmic envelope spectrum will be demonstrated.