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Fast Bayesian approach for modal identification using free vibration data, Part I – Most probable value

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ABSTRACT

The identification of modal properties from field testing of civil engineering structures is becoming economically viable, thanks to the advent of modern sensor and data acquisition technology. Its demand is driven by innovative structural designs and increased performance requirements of dynamic-prone structures that call for a close cross-checking or monitoring of their dynamic properties and responses. Existing instrumentation capabilities and modal identification techniques allow structures to be tested under free vibration, forced vibration (known input) or ambient vibration (unknown broadband loading). These tests can be considered complementary rather than competing as they are based on different modeling assumptions in the identification model and have different implications on costs and benefits. Uncertainty arises naturally in the dynamic testing of structures due to measurement noise, sensor alignment error, modeling error, etc. This is especially relevant in field vibration tests because the test condition in the field environment can hardly be controlled. In this work, a Bayesian statistical approach is developed for modal identification using the free vibration response of structures. A frequency domain formulation is proposed that makes statistical inference based on the Fast Fourier Transform (FFT) of the data in a selected frequency band. This significantly simplifies the identification model because only the modes dominating the frequency band need to be included. It also legitimately ignores the information in the excluded frequency bands that are either irrelevant or difficult to model, thereby significantly reducing modeling error risk. The posterior probability density function (PDF) of the modal parameters is derived rigorously from modeling assumptions and Bayesian probability logic. Computational difficulties associated with calculating the posterior statistics, including the most probable value (MPV) and the posterior covariance matrix, are addressed. Fast computational algorithms for determining the MPV are proposed so that the method can be practically implemented. In the companion paper (Part II), analytical formulae are derived for the posterior covariance matrix so that it can be evaluated without resorting to finite difference method. The proposed method is verified using synthetic data. It is also applied to modal identification of full-scale field structures.

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1. Introduction

Modern sensor and data acquisition technology have allowed structural responses to be practically measured with reasonable quality. The vibration data of an as-built structure can be used for identifying its 'modal properties', which primarily consist of the natural frequencies, damping ratios and mode shapes. The modal properties are indispensable for structural health monitoring, model updating and damage detection [1–3]. Three types of tests are commonly performed for obtaining vibration data for modal identification. In an ambient vibration test, vibration data is obtained when the structure is in its working condition without artificial loading [4–6]. The input ambient loading arising from the environment and operational activities is not measured but is assumed to be broadband random. Economy is a significant advantage, although the signal-to-noise (s/n) ratio and hence the identification possibility/quality cannot be directly controlled. In a forced vibration test, artificial known excitation is applied to the structure [7–10]. The vibration level and hence the s/n ratio can be directly controlled and enhanced significantly. It is often more expensive because special equipment such as a high-payload shaker is needed and greater risk management is required. In a free vibration test, vibration data is obtained when the structure is dominantly under free vibration. Although it still requires artificial excitation to initiate vibration of the structure to an adequate level, such excitation need not be measured and so it allows greater flexibility and puts less demand on equipment.

Many modal identification approaches using free vibration data have been developed. The Ibrahim time domain method (ITD) makes use of generalized eigenvalue decomposition [11–13]. It was originally developed for displacement response data but later generalized for velocity and acceleration data through a state-space formulation [11,13]. The complex exponential method makes use of singular value decomposition [14]. The least square method is conceptually straightforward and it identifies the set of modal parameters as the one that minimizes a measure of fit between the theoretical and measured response in a least square sense [15,16].

Proper orthogonal decomposition or the so-called Karhunen Loeve decomposition was used for statistical analysis of free vibration response data and identifying the mode shape vectors [17]. It was later applied to response data under free vibration and harmonic excitations [18]. It was also applied to obtain proper orthogonal modes under proportionality assumption on the mass matrix [19]. The constraint was later removed, leading to the method MAFVFO (Modal Analysis by using Free Vibration Response Only) [20]. Based on the measured data, other kinematic quantities (e.g., displacement, velocity, acceleration) consistent with the data are determined by numerical differentiation/integration and are subsequently used for modal identification. The method is applicable for discrete and continuous mass systems and it has been extended to treat non-proportional damping [21].

Methods based on wavelet transforms have also been developed. Discrete wavelet transform (DWT) was combined with the Hilbert transform to determine the natural frequencies and damping ratios of a structure [22]. It was later extended to analyze free vibration response and earthquake response, where the equations of motion were determined by DWT to identify the modal parameters corresponding to different kinds of mother wavelet functions [23]. One drawback of DWT is its computational inefficiency. In view of this, methods based on continuous wave transform (CWT) were developed for modal identification based on the modulus and phase of the transform [24–26]. The Hilbert–Huang transformation [27], a general method for analyzing nonlinear and non-stationary signal analysis, has also been applied to modal identification with free vibration data for linear and nonlinear systems [28,29].

Despite the abundance of the aforementioned methods, they are based on empirical statistical proxies (e.g., correlation functions) for identification and they do not account for the identification uncertainty of the modal parameters. The former implies that the method may not have utilized all information in the data for identification. The latter is especially relevant for field vibration data as there is less control on the test condition compared to laboratory data. In free vibration field tests, the measured data mainly consists of three parts: free vibration response, measurement noise and ambient vibration response. Existing identification methods often assume that the first two components dominate the measured data and ignore the effect of the last component. In reality, arising from the measurement noise and ambient component, the measured data exhibits a variety of spectral characteristics over its sampling bandwidth (i.e., up to the Nyquist frequency), many of which are irrelevant to the modes to be identified or are difficult to model. These issues call for a careful choice of assumptions in the identification model and a fundamental formulation to account for uncertainty. Otherwise it may lead to significant bias or conclusions inconsistent with structural dynamics.

In this work, a Bayesian statistical method based on the Fast Fourier Transform (FFT) of free vibration data is developed. It exploits the statistical properties of the FFT data to construct a posterior probability density function (PDF), providing a fundamental means for identifying the modal parameters as well as their uncertainty consistent with probability logic and modeling assumptions. As is common in Bayesian methods, computational difficulties are encountered when determining the posterior MPV and the associated covariance matrix. From first principle, determining the MPV requires solving a numerical optimization problem whose dimension grows linearly with the number of measured degrees of freedom (dofs). In view of this, computational strategies are developed by exploiting the mathematical structure of the problem in well-separated and closely-spaced mode situations. In the companion paper, the posterior covariance matrix of the modal parameters is investigated, where closed-form analytical expressions are derived for evaluation without resorting to finite difference method. Examples with synthetic data are also provided to verify the proposed theory. Field test data are used to illustrate the practical application of the method.