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Parameter selection and covariance updating

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ABSTRACT

A simple expression is developed for covariance-matrix correction in stochastic model updating. The need for expensive forward propagation of uncertainty through the model is obviated by application of a formula based only on the sensitivity of the outputs at the end of a deterministic updating process carried out on the means of the parameters. Two previously published techniques are show to reduce to the same simple formula within the assumption of small perturbation about the mean. It is shown, using a simple numerical example, that deterministic updating of the parameter means can result in correct reconstruction of the output means even when the updating parameters are wrongly chosen. If the parameters are correctly chosen, then the covariance matrix of the outputs is correctly reconstructed, but when the parameters are wrongly chosen is found that the output covariance is generally not reconstructed accurately. Therefore, the selection of updating parameters on the basis of reconstructing the output means is not sufficient to ensure that the output covariances will be well reconstructed. Further theory is then developed by assessing the contribution of each candidate parameter to the output covariance matrix, thereby enabling the selection of updating parameters to ensure that both the output means and covariances are reconstructed by the updated model. This latter theory is supported by further numerical examples.

1. Introduction

One of the first attempts to address the problem of updating or ‘correcting’ finite element models was the statistical approach proposed by Collins, Hart, Hasselman and Kennedy in 1974 [1]. Since that time much attention has been concentrated mainly on deterministic model updating methods, including particularly parameterisation of finite element models for updating and regularisation of the generally ill-posed model-updating problem. Details can be found in Refs. [2–4]. Very recently, new research has addressed the problem of stochastic model updating, which we review briefly in the following paragraphs.

Jacquelin et al. [5] developed a model updating technique using random matrix theory resulting in a mean stiffness and covariance matrix representing the structural uncertainty in a global way from measured variability in natural frequencies and modes shapes. Adhikari and Friswell [6] used a sensitivity approach to update distributed parameters, typically the

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bending rigidity *EI* of a beam, represented as random fields using the Karhunen–Loève expansion. Goller et al. [7] addressed the problem of insufficient information by the application of multi-dimensional Gaussian kernel densities derived from sparse modal data. This allowed design insensitivity to be quantified, so that the proposed method could be said to be robust. Mthembu et al. [8] used Bayesian evidence for model selection.

Early examples of Bayesian model updating include the work of Beck and Katafygiotis [9,10] whereby experimental data is used to progressively revise the updating parameters expressed by a posterior probability density function. One problem with the Bayesian approach has been the large computational effort associated with sampling using Markov chain Monte-Carlo (MCMC) algorithms. This has now been largely overcome as demonstrated by Goller et al. [11] using parallelisation of the updating code together with the transitional MCMC algorithm, which identifies parameter regions with the highest posterior probability mass. Zhang et al. [12] used the polynomial chaos expansion as a surrogate for the full FE model as well as an evolutionary MCMC algorithm where a population of chains is updated by mutation to avoid being trapped in local basins of attraction.

The problem of variability in the dynamics of nominally identical test pieces seems to have been addressed first by Mares et al. [13,14] using a multivariate gradient-regression approach. This was combined with a minimum variance estimator so that the means of the resulting distributions represented the most likely parameters of a next-tested structure and the standard deviations could be interpreted as indicators of confidence in the means. Hua et al. [15] were the first to consider the uncertainty of multiple nominally-identical test pieces from the frequentist viewpoint, where the distribution is meaningful in terms of the 'spread' of updating parameters. They used a perturbation approach, as did Haddad Khodaparast et al. [16], the latter showing excellent results using first-order perturbation whereas the method described in [15] required the computation of second-order sensitivities. Govers and Link [17] extended the classical sensitivity model-updating method by a Taylor series expansion of the analytical output covariance matrix and obtained parameter mean values and covariances. This technique has since been demonstrated very effectively, and compared to an interval updating method [18], using data obtained by repeated disassembly and reassembly of the DRL AIRMOD structure [19,20]. Fang et al. [21,22] used a response-surface surrogate for the full FE model together with Monte-Carlo simulation (MCS). Hypothesis testing by analysis of variance (ANOVA) using the statistical *F*-test evaluation was applied to determine the contribution of each updating parameter (or a group of parameters) to the total variance of each measured output. If the *F*-test returned a value that exceeded a threshold, then the chosen parameter was deemed to contribute significantly to the variance of the output.

In this paper, a simple formula is developed for covariance updating that can be applied without the use of expensive forward propagation by MCS to determine the output covariance matrix. Two previous stochastic model updating methods are shown to be equivalent to the same formula with the assumption of small perturbations about the mean. It is demonstrated using a 3-degree of freedom model that the choice of updating parameters is critical to this process. If the correct parameters are chosen, then the output covariance matrix is reconstructed faithfully. However, this is generally not the case when wrongly chosen parameters are used, even though the output means may be accurately reconstructed. It is shown that the scaled output covariance matrix may be decomposed to allow the contributions of each candidate parameter to be assessed. Use of the classical linearised sensitivity permits the assessment to be carried out efficiently. Numerical examples are used to illustrate the performance of the technique.

2. Updating the covariance matrix

The stochastic model updating problem may be expressed as,

$$\left(\mathbf{z}^e-\overline{\mathbf{z}}^e\right)=\overline{\mathbf{S}}_j\left(\boldsymbol{\theta}-\overline{\boldsymbol{\theta}}\right)_{j+1}+\boldsymbol{\varepsilon}_{j+1} \tag{1}$$

by the assumption of small perturbation about the mean. In Eq. (1) the over-bar denotes the mean, $\mathbf{z}^e$, $\overline{\mathbf{z}}^e$ are experimentally measured outputs, typically natural frequencies and mode-shape terms, $\boldsymbol{\theta}_{j+1}$ is the $(j+1)$th estimate of parameter distribution to be determined, with mean $\overline{\boldsymbol{\theta}}_{j+1}$. The mean sensitivity matrix is denoted by $\overline{\mathbf{S}}_j=\mathbf{S}\left(\overline{\boldsymbol{\theta}}_j\right)$ and $\boldsymbol{\varepsilon}_{j+1}$ represents errors introduced from various sources including inaccuracy of the model and measurement imprecision.

Model updating of the means is a deterministic problem [16,17] given by,

$$\overline{\boldsymbol{\theta}}_{j+1}=\overline{\boldsymbol{\theta}}_j+\overline{\mathbf{T}}_j\left(\overline{\mathbf{z}}^e-\overline{\mathbf{z}}_j^a\left(\overline{\boldsymbol{\theta}}_j\right)\right) \tag{2}$$

where $\overline{\mathbf{z}}_j^a\left(\overline{\boldsymbol{\theta}}_j\right)$ is the a predicted output of the model at the jth iteration. The transformation matrix $\overline{\mathbf{T}}_j$ is the generalised pseudo inverse of the sensitivity matrix $\overline{\mathbf{S}}_j$,

$$\overline{\mathbf{T}}_j=\left(\overline{\mathbf{S}}_j^T\mathbf{W}_\varepsilon\overline{\mathbf{S}}_j+\mathbf{W}_\vartheta\right)^{-1}\overline{\mathbf{S}}_j^T \tag{3}$$

and $\mathbf{W}_\varepsilon$ and $\mathbf{W}_\vartheta$ are weighting matrices, to allow for regularisation of ill-posed sensitivity equations [4].

It is seen from Eq. (1) that the matrix of output covariances is given by,

$$Cov\left(\Delta\mathbf{z}^e,\Delta\mathbf{z}^e\right)=\overline{\mathbf{S}}_jCov\left(\Delta\boldsymbol{\theta}_{j+1},\Delta\boldsymbol{\theta}_{j+1}\right)\overline{\mathbf{S}}_j^T+Cov\left(\boldsymbol{\varepsilon}_{j+1},\boldsymbol{\varepsilon}_{j+1}\right) \tag{4}$$