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# Partial pole placement by feedback control with inaccessible degrees of freedom

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ABSTRACT

Classical pole-placement theory requires that every degree of freedom shall be accessible to sensing but in physical systems there are often obstructions that make sensing at certain degrees of freedom impractical. In the classical formulation of the pole placement problem the input vector which determines the actuator gains is given and the pole placement problem is linear. If the input vector is not known and it is desired to find the gains of actuators and the gains of the measured state subject to some constraints then the problem becomes nonlinear since the unknown parameters multiply each other. It is shown that this nonlinear active vibration control problem is rendered linear by the application of a new double input control methodology implemented in conjunction with a receptance-based scheme where full pole placement is achieved while some chosen degrees of freedom are free from both actuation and sensing. A lower bound on the maximum number of degrees of freedom inaccessible to both actuation and sensing is established. A numerical example is provided to demonstrate the working of the method using the new double-input approach.


## 1. Introduction

The eigenvalue assignment problem has many potential applications in structural dynamics, including the improvement of stability of dynamic systems, avoidance of the damaging large-amplitude vibrations close to resonance, and adaptive changes to system behaviour. A variety of eigenvalue assignment algorithms have been developed over several decades, namely, eigenvalue assignment by state feedback [1,2], eigenvalue assignment by output feedback [3–8], robust eigenvalue assignment problem [9–15], to name but a few. In practice, there may be a large number of eigenvalues but only a few that are undesirable. Therefore, partial pole placement, where some eigenvalues are required to be relocated and the remaining poles are rendered unchanged, is of practical value in suppressing vibration and stabilising dynamic systems. Saad [16] proposed a projection algorithm for the partial eigenvalue assignment for first-order systems. Datta et al. [17] developed a closed-form solution to the partial pole assignment problem by state feedback control in systems represented by second order differential equations. The method has been generalised for the case of multi-input control [18,19]. Chu [15] proposed a partial pole assignment method with state feedback for second-order systems. The robust partial pole placement problem was investigated in [20–23]. The problem of optimising the control effort in partial eigenvalue assignment was addressed by Guzzardo et al. [24]. Partial pole placement with time delay was also considered [25–27].

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Ram and Mottershead [28] developed a new theory known as the receptance method for eigenvalue assignment in active vibration control using experimental measurements. While in conventional pole placement methods analytical models are required, in the method of receptances, measured modal data are used instead of system matrices. Therefore, the receptance method has a wealth of advantages. There is no need to estimate or know the mass, stiffness and damping matrices, no need to estimate the unmeasured state using an observer or a Kalman filter and no need for model reduction. By virtue of partial controllability, a partial pole placement approach using measured receptances for single-input and multi-input feedback control was proposed by Tehrani et al. [29]. Very recently, Ram and Mottershead [30] developed a new theory of receptance-based partial pole placement by using partial observability. A series of experimental tests were carried out to demonstrate the capability of the receptance method in active vibration suppression [29,31–33].

In the traditional application of active vibration control by partial pole placement with state feedback the input vectors are assumed to be given and the calculated vectors of the control gain are therefore in general fully populated. Consequently, to realise the control in practice it is required to sense the state at each degree of freedom. In applications, however, some of the degrees of freedom may not be physically accessible to actuation and sensing simultaneously. That is, there exist some inaccessible degrees of freedom.

The purpose of the paper is quite different from sparse controllability problem [34,35] whereby optimisation is applied to ensure controllability (and separately observability) with as few variables as possible, leading the fewest total number of sensors and actuators. The present work is motivated by engineering practicality where not every degree of freedom is available for sensing, and if it is not available for sensing it is not available for actuation either. One example is the helicopter rotor blade requiring active vibration control but inaccessible to both actuation and sensing.

In the present paper the input and control-gain vectors are determined and the resulting interactions between unknown terms that would normally lead to nonlinearity are circumvented by the use of a new double input control involving position, velocity and acceleration feedback. This enables the retained modes to be separated into two sets resulting in a linear system of constrains. Further constraints are applied to assign the other modes and it is seen that the nonlinear problem of determining input vectors and the control gains for partial pole placement with inaccessible degrees of freedom is converted into a linear one. A lower bound on the maximum number of degrees of freedom completely cleared of both sensing and actuation is then established using purely linear analysis. Since the main objective is the introduction of a new concept, we address for simplicity the case involving distinct eigenvalues in both open and closed loops. Systems with repeated eigenvalues will be considered in further work beyond the scope of the present article.

Section 2 of this paper establishes the basis for the analysis that follows. In Sections 3 and 4 the necessary equations are established for partial pole placement with inaccessible actuators and sensors represented by zero terms in the input vector and the control-gain vectors. Section 5 establishes the solvability conditions that enable lower bounds on the maximum numbers of inaccessible actuators and sensors to be determined. Then in Section 6 a lower bound on the maximum number of degrees of freedom inaccessible to both actuation and sensing is achieved by equating the solutions obtained in the previous section. A numerical example is used to demonstrate the working of the proposed theory.

## 2. Motivation

The motion of the $n$ degree of freedom system.

$$\mathbf{M}\ddot{\mathbf{x}}+\mathbf{C}\dot{\mathbf{x}}+\mathbf{K}\mathbf{x}=\mathbf{0}, \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are symmetric $n \times n$ matrices and where $\mathbf{M}$ is positive-definite and $\mathbf{C}$ and $\mathbf{K}$ are positive-semidefinite, may be altered by state feedback control.

$$\mathbf{M}\ddot{\mathbf{x}}+\mathbf{C}\dot{\mathbf{x}}+\mathbf{K}\mathbf{x}=\mathbf{b}u(t) \tag{2}$$

where

$$u(t)=\mathbf{f}^T\dot{\mathbf{x}}+\mathbf{g}^T\mathbf{x} \tag{3}$$

and where $\mathbf{b}$, $\mathbf{f}$ and $\mathbf{g}$ are real vectors denoting force-distribution and control-gain terms.

The quadratic eigenvalue problem corresponding to the open loop system (1) is given by

$$(\lambda_k^2\mathbf{M}+\lambda_k\mathbf{C}+\mathbf{K})\mathbf{v}_k=\mathbf{0}, \quad k=1,\ 2,\dots,\ 2n. \tag{4}$$

The self-conjugate set of $2n$ poles, $\{\lambda_k\}_{k=1}^{2n}$, with corresponding eigenvectors $\{\mathbf{v}_k\}_{k=1}^{2n}$ that satisfy (4) are the eigenpairs of the open-loop system.

Similarly, the eigenvalue problem of the closed loop system (2) is

$$\left(\mu_k^2\mathbf{M}+\mu_k\left(\mathbf{C}-\mathbf{b}\mathbf{f}^T\right)+\mathbf{K}-\mathbf{b}\mathbf{g}^T\right)\mathbf{w}_k=0, \quad k=1,\ 2,\dots,\ 2n. \tag{5}$$

with the self-conjugate set of $2n$ poles, $\{\mu_k\}_{k=1}^{2n}$, and corresponding eigenvectors $\{\mathbf{w}_k\}_{k=1}^{2n}$. The eigenvalues of the open-loop system are assumed to be distinct, as are those of the closed-loop system, the case of repeated roots and defective systems is to be considered in further work beyond the scope of the present article.