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Degradation reliability modeling based on an independent increment process with quadratic variance

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ABSTRACT

Degradation testing is an important technique for assessing life time information of complex systems and highly reliable products. Motivated by fatigue crack growth (FCG) data and our previous study, this paper develops a novel degradation modeling approach, in which degradation is represented by an independent increment process with linear mean and general quadratic variance functions of test time or transformed test time if necessary. Based on the constructed degradation model, closed-form expressions of failure time distribution (FTD) and its percentiles can be straightforwardly derived and calculated. A one-stage method is developed to estimate model parameters and FTD. Simulation studies are conducted to validate the proposed approach, and the results illustrate that the approach can provide reasonable estimates even for small sample size situations. Finally, the method is verified by the FCG data set given as the motivating example, and the results show that it can be considered as an effective degradation modeling approach compared with the multivariate normal model and graphic approach.

1. Introduction

Reliability and safety are of significant importance for long-life and highly reliable products, which are generally required to work with a good performance state even after long periods of time. Considering the conventional method, ample failure data has to be obtained through internal life tests or accelerated life tests as the life information basis for reasonable assessment [1]. However, sometimes, no failures occur during such tests for those systems, which are developed to last longer and perform more reliably. The reason is that their life test time has to be limited due to the short product-development time and company's profit maximizing requirements [2].

Because performance degradation measures taken over test time contain lifetime information for many components, an alternative approach for assessing the failure of a device is to grasp the changes in performance that cause its failure [3]. By defining failure in terms of a specified level of degradation, degradation analysis can establish the relationship between the reliability index and performance feature, and the FTD can be obtained accordingly. Reliability analysis based on degradation

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data illustrates many significant advantages. For example, the deterioration test does not need to last until failure occurs and less acceleration is adopted in degradation data collection [4].

Over the past two decades, degradation tests have attracted much interest and become popular techniques in engineering, especially for the reliability analysis of highly reliable products [5]. Numerous models have been developed and applied to specific degradation tests [6,7]. The general degradation path method and stochastic process approach, the ones most commonly seen in the literature, are worth noting and discussing. General degradation path methodology is defined as a mixed-effects model by Yuan and Pandey [8] and as a sample-based technique by Mohammadian et al. [9]. In its modeling process, specimens are assumed to illustrate the same degradation path function form, and the values of some or all of the model parameters can vary from unit to unit. Two main stages, including degradation path selection for all items and parameter determination for each unit, are involved in the processing procedure. General references for this approach are Lu and Meeker [10]; Lu et al. [11]; Meeker and Escobar [12]; Pettit and Young [13] and Peng and Tseng [14]. In practical application, however, the independence of the successive measurement for an individual item cannot be properly considered and depicted, and ample sample size is needed to guarantee the analysis precision because of information loss [8].

On the other hand, a degradation procedure can be taken as a certain stochastic process because of the generally complex degradation mechanisms. Two main methodologies are worth discussing. One is a two-stage technology based on the non-stationary stochastic process theory. It is called graphic method by Nelson [15] and is similar to the interval-based technique in Mohammadian et al. [9]. The two main steps involved are as follows: (1) A uniform distribution type (such as normal distribution) has to be selected to properly depict the measurements' statistical properties at each test time, and then the distribution parameters can be estimated; and (2) One fits the individual parameter estimation results as functions of test time through regression. Taking normal distribution as an example, mean and variance estimates can be obtained by implementing a statistical analysis on the performance measurements at each specific test time. Then, mean and variance functions can be constructed to exhibit the time-varying regulation of their statistical properties. Consequently, life-time information can be inferred and reliability index can be constructed. The most significant advantage of this two-stage method is that it can be extended from normal distribution to others, such as Weibull distribution [16] or a general non-parametric distribution [17]. As a two-stage processing procedure, however, abundant measurements at each observation time are required to guarantee the analysis accuracy [18].

The other methodology treats degradation procedures as certain stochastic processes with one-stage or two-stage parameter estimations. Because the Lévy process provides a very general framework for modeling a wide variety of stochastic models, the compound Poisson process, Wiener process and gamma process are discussed with regard to reliability engineering. A compound Poisson process is a stochastic process with independent and identically distributed jumps that occur according to a Poisson process [19]. It is a jump process that has a finite number of jumps in finite time intervals and is, therefore, suitable for modeling usage damage due to sporadic shocks. Regarding degradation analysis, it is often used to model degradation due to discrete shocks. In our study, however, we focus on peaceful degradation processes when discussing unmaintainable systems with stable intensity of use.

The Wiener process is also called Brownian motion. Brownian motion with drift is a stochastic process $\{X(t), t \geq 0\}$ with independent, real-valued increments and decrements that illustrate a normal distribution with mean μt and variance $\sigma^2 t$ for all $t \geq 0$. To analyze deterioration in various situations, multiple models, including the linear degradation model [14], Bayesian method [20] and integrated methodology [21], have been constructed. According to Wang [22], a well-adopted form for the regular Wiener process $\{X(t), t \geq 0\}$ in degradation analysis can be expressed as $X(t) = \beta\Lambda(t) + \sigma W(\Lambda(t))$, where $\Lambda(t)$ is called the transformed time scale. The Wiener process has s-independent and normally distributed increments; i.e.,$\Delta X(t) = X(t + \Delta t) - X(t)$ is s-independent of $X(t)$ and $\Delta X(t) \sim N(\beta\Lambda(t + \Delta t) - \beta\Lambda(t), \sigma^2\Lambda(t + \Delta t) - \sigma^2\Lambda(t))$. Consequently, for the commonly adopted linear degradation path (mean degradation has a linear relationship of transformed time $E(X(t)) = \beta\Lambda(t)$, where $\Lambda(t)$ illustrates a linear form), we can conclude, based on the preceding model, that the variance of $X(t)$ also illustrates a linear function form of transformed time $\mathrm{Var}(X(t)) = \sigma^2\Lambda(t)$. However, many practical linear degradation processes exhibit quadratic variances and cannot be properly depicted by the previous well-adopted form for the regular Wiener process. In recent literature, the popular Wiener degradation process model incorporating nonlinear structures has been investigated for diagnostics, prognostics remaining useful life estimation [23–29].

A gamma process is a stochastic process with independent, non-negative increments that have a gamma distribution with an identical scale parameter. Let $\{X(t), t \geq 0\}$ denote a gamma process with a shape function $v(t) > 0$ (where $v(t)$ is a non-decreasing, right-continuous, real valued function for $t \geq 0$ with $v(0) = 0$) and a scale parameter $u > 0$; then, expectation and variance can be given by $E(X(t)) = v(t)/u$ and $\mathrm{Var}(X(t)) = v(t)/u^2$. Although the random effects model [30], non-stationary model [31], increasing pure jump process model [32], and possibly time-dependent covariates model [33] have been established for various practical applications, the similar problem (expectation and variance will illustrate the same function form) cannot be ignored. This means the widely adopted linear degradation path will lead to a linear variance. Another critical issue one should note is that tremendous challenges will be encountered when we want to obtain the closed-form form of a PDF with the gamma process based model because digamma functions are involved in the analysis procedure [34].

In our previous study [35] discussing unmaintainable systems of stable intensity of use, we declare a unit as having failed when its degradation reaches a critical threshold [4]. It was further supposed that the measurements reflect the inherent randomness of degradation itself. Degradation processes of this situation were discussed and a degradation analysis approach of an independent increment process with a linear mean function and linear standard deviation function of