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Chatter detection in turning using persistent homology

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ABSTRACT

This paper describes a new approach for ascertaining the stability of stochastic dynamical systems in their parameter space by examining their time series using topological data analysis (TDA). We illustrate the approach using a nonlinear delayed model that describes the tool oscillations due to self-excited vibrations in turning. Each time series is generated using the Euler-Maruyama method and a corresponding point cloud is obtained using the Takens embedding. The point cloud can then be analyzed using a tool from TDA known as persistent homology. The results of this study show that the described approach can be used for analyzing datasets of delay dynamical systems generated both from numerical simulation and experimental data. The contributions of this paper include presenting for the first time a topological approach for investigating the stability of a class of nonlinear stochastic delay equations, and introducing a new application of TDA to machining processes.

1. Introduction

Deterministic models for chatter in machining dynamics have been the subject of extensive research in recent years [1]. However, machining processes are inherently stochastic with many noise sources due, for example, to the variation of the material parameters [2,3], external noise sources [4], and variations of the delay [5]. Thus, the scarcity of analysis tools for stochastic delay systems remains a fundamental impediment to the continuous progress in the metal removal industry [6,7].

Stochastic equations are infinite dimensional and therefore analysis tools from deterministic models are not readily applicable to them. The analysis is more challenging if the dynamics involve delays so that the system model is a stochastic delay differential equation SDDE). Since machine tool chatter is typically described by these difficult systems, the number of studies on stochastic machining dynamics remains small, particularly in comparison to its deterministic counterpart. We note here that in addition to machining dynamics [3,4], SDDEs arise in many applications such as chemical kinetics [8] and genetic networks [9]. Therefore, developing or extending analytical and numerical tools for their analysis continues to be an active and important area of research.

For a limited number of SDDEs, stochastic calculus can be used to study the stability of the first and second moments [10]. If the delay is small, then the SDDE can be approximated using a stochastic differential equation without the delay term [11]. An extension of the semi-discretization method for studying the moment stability of linear SDDEs with delays appearing in the drift term only was described in [12]. Another method to investigate the stability of this class of equations

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uses the Lyapunov approach [13]. However, for the general case of SDDEs numerical simulation remains the most viable method of analysis.

Euler–Maryuama and Milstein simulation methods, originally used to simulate stochastic differential equations, were extended to SDDEs in [14–17] and [18], respectively. Numerical simulation provides a tool for generating path-wise solutions that can be easier to investigate than the original SDDE. For example, instead of directly studying the mean square (or more generally, the pth mean) stability of the SDDE which might be difficult or impossible, the paths generated by numerical simulation can be used [19–21]. The result of the numerical simulation is a time series or dataset that contains information about the system dynamics. Developing data analysis tools for these datasets has two benefits: (1) it provides a benchmark for testing new methods for the analysis of SDDE, and (2) the same tools can be used to analyze data from real-world applications, e.g. machining dynamics.

Some of the data analysis methods for non-delayed stochastic equations include principal component analysis [22,23], multi-dimensional scaling [24], local linear embedding [25], Laplacian eigenmaps [26], Hessian eigenmaps [27], local tangent space alignment [28], and diffusion maps [29–31]. The first step in many of these methods is to obtain a lower-dimensional representation of the underlying high-dimensional manifold. The hope is that the simplified representation captures the main features of the underlying dynamics. One of the key assumptions in many of the prominent data analysis methods for dynamical systems, e.g. diffusion maps, is that the underlying dynamics is Markovian. This precludes them from being used to study SDDEs which are non-Markovian.

Frequency domain methods, typically based on Fourier transform and the power spectrum, have also been used for data analysis especially when the signal is periodic. However, when the signal is non-periodic, the leakage effects inherent in Discrete Fourier Transform (DFT) will result in errors unless the input is periodic and the length of the time series is equal to the input's period [32]. Further, in the context of machining we are interested in changes in the time series associated with the change of the cutting process from stable to unstable; however, frequency domain analysis does not account for changes in the signal with time. Other issues with the frequency domain analysis include the limitations in analyzing trending time series and data with quasi-periodic motion [33]—a common route to chaos which is often associated with unstable cuts [34]. In order to address these limitations, some success has been reported using time-frequency methods [35] and wavelets [36]. However, the topic of analyzing stochastic machining models remains an active area of research.

In this paper we explore data analysis tools for studying the stability of a stochastic turning model using topological data analysis. These tools are applicable to datasets arising from both experiments as well as simulations of dynamical systems. Specifically, we will use persistent homology [37–40] to automatically detect when changes in the system behavior indicative of chatter occur near the stability boundary of the linearized, noise-free model. In contrast to other data analysis tools, persistent homology does not attempt to obtain a lower dimensional representation of a data set, but rather a low-dimensional descriptor which is easy to understand and which can be used to find and measure properties of interest.

Persistent homology has found success in applications to many diverse fields such as neuroscience [41,42], genetics [43,44], epidemiology [45], tracking [46,47], international relations [48], map reconstruction [49], sensor networks [50,51], and image analysis [52]. Most recently, a great deal of work has looked at using persistence for signal analysis. The idea behind this method is to use the Takens embedding [53] to turn a signal into a point cloud in high dimensional space and analyze the resulting point cloud using persistence. In [54], it was shown that this method provides a framework to parameterize dynamics; the procedure is also amenable to combination with machine learning for automation [55]. Variations of this idea along with broader applications of topology have been used to study signals from human speech [56], wheezing in breathing signals [57], gene expression data [58–61], computer architecture [62,63], and character animation [64]. See also Chap. 6 of [65] or [66] for the formulation of the procedure using cohomology.

The standard Takens theorem assumes that the system is deterministic and autonomous [53]. However, the theorem has been extended to deterministic non-autonomous equations and to stochastic systems [67–69], as well as to infinite-dimensional dynamical systems [70]. Since the focus of this paper is on describing a topological approach for studying nonlinear stochastic machining models, we utilize the standard Takens theorem with the understanding that embedding timesets with large stochastic terms is likely to be unsuccessful. The extension of Takens theorem to nonlinear delay models with large stochastic terms is outside the scope of this paper and it is a topic of active research.

We demonstrate the main concepts using a second order nonlinear stochastic delay equation with multiplicative noise that models a single degree of freedom turning process. To keep the analysis clear, we only introduce one random component using a stochastic cutting force coefficient where the source of stochasticity can be the variation in the temperature, shear angle, or workpiece material properties. In particular, we study the model SDDE in the space of the non-dimensional spindle speed and depth of cut, and we use Euler–Maruyama method to simulate the SDDE for different combinations of these two process parameters. We show that as the value of the delay is varied as a result of varying the spindle speed, persistent homology can be used to detect the change of the response from a steady state equilibrium to a periodic orbit indicating the loss of stability through a Hopf bifurcation. We also demonstrate the effect of the noise intensity on the stability diagrams.