Mechanical Systems and Signal Processing 70-71 (2016) 613–624

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Identification of a finite number of small cracks in a rod using natural frequencies

E.I. Shifrin *

A.Yu. Ishlinsky Institute for Problems in Mechanics, Russian Academy of Sciences, Prospect Vernadskogo 101-1, Moscow, Russia

ARTICLE INFO

Article history:
Received 7 May 2015
Received in revised form
29 July 2015
Accepted 10 September 2015

Keywords:
Rod
Longitudinal vibration
Natural frequencies
Small cracks
Inverse problem

ABSTRACT

A problem of identification of a finite number of small cracks in a rod by means of natural frequencies corresponding to longitudinal vibration is considered. The cracks are simulated by massless translational springs. It is known, that generally for unique identification of multiple cracks in a rod it is necessary to know two spectra. An algorithm is developed enabling in case of small cracks to extract an important information about the possible number of cracks, their positions and the flexibilities of the corresponding springs by means of only one spectrum. The developed method is extended for identification small multiple cracks in a simply supported beam.

1. Introduction

An inverse spectral problem for a rod with a finite number of transverse cracks is considered. The cracks are simulated by massless translational springs. It is assumed that the cracks are small. It denotes that the flexibilities of the springs, simulating the cracks are small. More accurate definition of the smallness of cracks is given below. It is assumed also that such a number is known, which limits the number of cracks above. In the recent paper [1] it was proved that for unique identification of a finite number of cracks it is necessary to know two spectra corresponding to longitudinal vibration. The suppositions, mentioned above, lead to significant simplification of the problem and enable to extract an important information about the cracks by means of a single spectrum. As we consider the case of small cracks, let us mention only some publications that take into account simplifications related to the smallness of cracks. Morassi [2] studied the changing of the natural vibration frequencies of rods and beams caused by the presence of small cracks. Narkis [3], Morassi [4] and Dilena and Morassi [5] considered the problem of identification of a single small crack in rods and beams. The problem of identification of two small cracks in rods and simply supported beams was considered in the papers of Morassi and Rollo [6] and Rubio et al. [7]. Khiem and Toan [8] proposed a scanning method for identifying of a finite number of small cracks in a beam. The method has led to good results in several considered examples. At the same time, it is necessary to note that scanning method is not rigorously justified and the limits of its applicability have not been established.

In the present paper we propose a simple, well justified method for extraction of an important information about the small multiple cracks in a rod by means of a single spectrum. The mathematical formulation of the considered problem is given in Section 2. A method of cracks reconstruction using a spectrum corresponding to free–free end conditions is given in

* Tel.: +7 4954343665; fax: +7 4997399531.
E-mail address: shifrin@ipmnet.ru

http://dx.doi.org/10.1016/j.ymssp.2015.09.023
0888-3270/

Section 3. A method of cracks identification using a spectrum corresponding to fixed-free end conditions is given in Section 4. An extension of the developed method on the case of simply supported beam is presented in Section 5. Numerical examples are considered in Section 6. The sensibility of the results to the noise in data studied in Section 7. Conclusions are presented in Section 8.

2. Statement of the problem

Let us consider a rod of length l and cross-section of constant area A. We assume that the rod occupies an interval $0<x<l$ and the translational springs, which simulate the localized damages, are located at points $x_1, x_2, \cdots, x_n$ such that $0=x_0<x_1<x_2<\cdots<x_n<x_{n+1}=l$. Denote by $u_j(x)$ the amplitudes of longitudinal displacements under time-harmonic vibration on the interval $x_{j-1}<x<x_j$, where $j=1,2,\cdots,n+1$. The equation of harmonic longitudinal oscillations has the following form, see [9,10]

$$u''_j(x)+\lambda u_j(x)=0, \quad j=1,2,\cdots,n+1, \quad x_{j-1}<x<x_j \tag{1}$$

where $\lambda=\rho\omega^2/E$, E is Young's modulus, ρ is the material density, ω is a circular frequency.

The conjugation conditions at the locations of springs are of the form, see [9, 10]

$$u'_j(x_j)=u'_{j+1}(x_j), \quad u_{j+1}(x_j)-u_j(x_j)=\Delta_j=EAc_j u'_j(x_j), \quad j=1,2,\cdots,n \tag{2}$$

where c_j is the flexibility of the jth translational spring.

We will consider two types of end conditions. The free–free condition has the form

$$u'_1(0)=0, \quad u'_{n+1}(l)=0 \tag{3}$$

The fixed-free end condition has the form

$$u_1(0)=0, \quad u'_{n+1}(l)=0 \tag{4}$$

Let us denote the eigenvalues of the problem (1), (2), (3) (except $\lambda=0$) by $\lambda_1, \lambda_2, \lambda_3, \cdots$ and the eigenvalues of the problem (1), (2), (4) by $\mu_1, \mu_2, \mu_3, \cdots$. The eigenvalues of the considered problems for a rod without cracks we denote by $\lambda_1^0, \lambda_2^0, \lambda_3^0, \cdots$ and $\mu_1^0, \mu_2^0, \mu_3^0, \cdots$, respectively. Let us denote $\sqrt{\lambda_k}=\xi_k$, $\sqrt{\lambda_k^0}=\xi_k^0$, $\sqrt{\mu_k}=\eta_k$, $\sqrt{\mu_k^0}=\eta_k^0$, $k=1,2,\cdots$. We suppose that such a number N is known that true number of cracks n satisfies the inequality $n\le N$. Below it is shown that small cracks can be reconstructed (in some sense) by $2N$ eigenvalues corresponding to free–free end conditions or $2N+1$ eigenvalues corresponding to fixed-free end conditions. In the case of free–free end conditions we assume that the cracks are small in the following sense:

$$EA|c|\sqrt{\lambda_{2N}^0}\ll 1, \quad |c|^2=\sum_{i=1}^{n} c_i^2 \tag{5}$$

In the case of fixed-free end conditions we suppose that the cracks are small when the following conditions are valid

$$EA|c|\sqrt{\mu_{2N+1}^0}\ll 1 \tag{6}$$

3. Cracks reconstruction using eigenvalues corresponding to free–free end conditions

Let us denote $\bar{c}_i=EAc_i$, $i=1,2,\cdots$. It was shown in Ref. [1] (see also [10]), that in case of N cracks the values $\xi_1, \xi_2, \cdots$, corresponding to free–free end conditions, are the roots of the following equation:

$$\det(\mathbf{Q})=0 \tag{7}$$

Here $\mathbf{Q}$ is a $(N+1)\times(N+1)$ matrix with the following elements:

$$\begin{aligned} &Q_{1k}=\xi \sin \xi(l-x_{k-1}), \quad k=1,2,\cdots,N+1, \quad x_0=0 \\ &Q_{pq}=\bar{c}_{p-1}\xi \sin \xi(x_{p-1}-x_{q-1}), \quad p=2,3,\cdots,N+1, \quad 1\le q<p \\ &Q_{pp}=1, \quad Q_{pq}=0, \quad p<q\le N+1 \end{aligned} \tag{8}$$

Because, according to our suppositions, the values $\bar{c}_i\xi_{2N}^0$ are small and the solutions of Eq. (7) $\xi_1, \cdots, \xi_{2N}$ are close to the values $\xi_1^0, \cdots \xi_{2N}^0$, respectively, consider several terms of the Taylor expansion of the function $\det Q(\bar{c}_1, \cdots, \bar{c}_N, \xi)$ in the neighborhood of the point $\left(0, \cdots, 0, \xi_k^0\right)$. Denote $\bar{c}=(\bar{c}_1, \cdots \bar{c}_N)$. The Taylor expansion is as follows:

$$|Q(\bar{c},\xi_k)|=|Q\left(0,\xi_k^0\right)|+\sum_{i=1}^{N}\frac{\partial|Q\left(0,\xi_k^0\right)|}{\partial c\backslash-t\backslash-t-_i}\bar{c}_i+\frac{\partial|Q\left(0,\xi_k^0\right)|}{\partial\xi}\Delta\xi_k+o\left(\sqrt{|\bar{c}|^2+\Delta\xi_k^2}\right)=0 \tag{9}$$