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Shaping frequency response of a vibrating plate for passive and active control applications by simultaneous optimization of arrangement of additional masses and ribs. Part I: Modeling

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ABSTRACT

An ability to shape frequency response of a vibrating plate according to precisely defined demands has a very high practical potential. It can be applied to improve acoustic radiation of the plate for required frequencies or enhance acoustic isolation of noise barriers and device casings by using both passive and active control. The proposed method is based on mounting severaladditional ribs and masses (passive and/or active) to the plate surface at locations followed from an optimization process. This paper, Part I, concerns derivation of a mathematical model of the plate with attached elements in the function of their shape and placement. The model is validated by means of simulations and laboratory experiments, and compared with models known from the literature. This paper is followed by a companion paper, Part II, where the optimization process is described. It includes arrangement of passive elements as well as actuators and sensors to improve controllability and observability measures, if active control is concerned.

1. Introduction

Vibrating plates can be used as structural sound sources, highly resistant to unfavourable environmental conditions. On the other hand, they can be used as noise barriers, which limit the acoustic energy transmission. In the latter case the vibration can be appropriately controlled with the aid of actuators to enhance their noise isolation. However, they exhibit complex frequency response, which can make it difficult to achieve satisfactory performance. Therefore, for both kinds of applications, it would be great to have an ability to shape the frequency response as desired. By locating resonance modes at excited frequencies the sound power radiated could be significantly increased [1–4]. In case of noise barriers, by relevant shaping of the frequency response a higher passive attenuation could be obtained or active control ability could be improved [5,6]. Frequency response shaping can also enhance efficiency of energy harvesting from vibrating structures [7].

In the literature there are no reported methods for precise shaping of the plate frequency response in strict accordance with the requirements, i.e. relocating or creating resonances and anti-resonances for selected frequencies. There are only works, which analyze the influence of additional concentrated masses or ribs on the plate frequency response [8,9]. Both translational and rotational kinetic energies have been considered [10,11]. Likewise, the effect of mounting ribs on vibration and acoustic radiation of

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Nomenclature

Roman Symbols

a	length of the plate
$A_{r,i}$	cross-sectional area of the i-th rib
$\mathbf{A}$	state matrix
b	width of the plate
$\mathbf{B}$	control matrix
$\mathbf{C}$	output matrix
$\mathbf{D}$	feedthrough matrix
D_x, D_y, D_{xy}	bending/twisting rigidities of the plate
E_x, E_y	Young moduli of the plate
$E_{r,i}$	Young modulus of the i-th rib
f_i	force generated by a i-th actuator
G_{xy}, G_{xz}, G_{yz}	shear moduli of the plate
$G_{r,i}$	shear modulus of the i-th rib
h	plate thickness
$I_{ax,i}$, $I_{sx,i}$, $I_{mx,i}$, $I_{ay,i}$, $I_{sy,i}$, $I_{my,i}$	moments of inertia of the i-th actuator, sensor and additional
$I_{r,i}$	second moment of inertia about the plate mid-plane of the i-th rib
$J_{r,i}$	torsional constant of the i-th rib
k_{tx0}, k_{tx1}, k_{ty0}, k_{ty1}	translational spring constants
k_{rx0}, k_{rx1}, k_{ry0}, k_{ry1}	rotational spring constants
$k_{r,i}$	radius of gyration of the i-th rib
$\mathbf{K}$	stiffness matrix
$m_{a,i}$, $m_{s,i}$, $m_{m,i}$	mass of the i-th actuator, sensor and additional mass, respectively
$\mathbf{M}$	mass matrix
N	number of Ritz functions
N_a, N_s, N_m, N_r	number of actuators, sensors, additional masses and ribs
$\mathbf{q}$	generalized plate displacement vector
$\mathbf{p}_x$, $\mathbf{p}_y$	generalized plate rotations vectors
$\mathbf{Q}$	vector of generalized forces
t	time
T	overall kinetic energy of the system
T_m, T_p, T_r	kinetic energy of additional masses, plate and ribs
$\mathbf{u}$	control vector
U	overall potential energy of the system
U_b, U_p, U_r	potential energy corresponding to elastic boundary restrains, plate and ribs
$\mathbf{v}$	modal displacement vector
$w(x, y, t)$	plate transverse displacement
$\mathbf{W}_c$, $\mathbf{W}_o$	Gramian matrices of controllability and observability, respectively
$\mathbf{y}$	output vector
i, j, k	positive integers
(x, y)	global coordinates
$(\tilde{x}_{r,i}, \tilde{y}_{r,i})$	local coordinates, corresponding to i-th rib
(ξ, η)	non-dimensional global coordinates
α_p	non-dimensional parameter equal $\frac{a}{b}$
$\beta_{r,i}$	shape factor of the i-th rib
$\Theta_x(x, y, t)$, $\Theta_y(x, y, t)$	cross-sectional rotations of the plate
κ_x, κ_y	shear coefficients of the plate
ν_x, ν_y	Poisson ratios of the plate
$\boldsymbol{\Xi}$	Damping matrix
$\xi_{d,i}$	Damping ratio of i-th eigenmode
ρ_p	mass density of the plate material
$\rho_{r,i}$	mass density of the i-th rib
$\boldsymbol{\phi}$, $\boldsymbol{\psi}_x$, $\boldsymbol{\psi}_y$	trial functions vectors
$\boldsymbol{\Phi}$	eigenvector matrix
$\boldsymbol{\Omega}$	eigenfrequency matrix
ω_i	i-th eigenfrequency

the plate has been investigated [12–15]. Different shapes and placements have been discussed [16]. General rules are known – additional masses lower the natural frequencies of the plate, whereas ribs elevate them. This can be easily observed in simulations with softwares for modal analysis. However, presence of additional masses and ribs has not been analyzed and used together, especially for shaping the frequency response according to precisely defined demands.

In this work it is proposed to simultaneously optimize arrangement of additional masses and ribs in order to reach precisely defined desired properties of the plate. The method has been submitted by the authors as a patent application [17]. The limits are related mainly to the maximum dimensions and mass of the created structure. Moreover, this approach can be employed for active structural control applications to reduce vibration and/or noise as well [18]. In such case, actuator and sensors bonded to the plate are modeled as additional masses of imposed shapes and weights. However, they are distinguished from the passive masses in notation, because locations of these elements determine the controllability and observability of the system [19,20]. In this paper, accelerometers and inertial actuators are considered, as examples of sensors and actuators, due to low costs of the elements themselves and their supporting electronics. However, this general idea can easily be applied for other kinds of sensors and actuators. Such sophisticated optimization, and shaping properties of the structure according to demands of various character followed from defined cost functions and constraints, would not be possible with available softwares for modal analysis.

The work is reported in the form of two papers. The current one, Part I, concerns the modeling, and the following paper, Part II, describes the optimization problem.

This paper is organized as follows. Section 2.1 is devoted to formulating the total energy functional of the system. In Section 2.2 the equation of the vibrating structure is derived. In Section 2.3 the state space model representation is