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Interpolated DFT algorithms with zero padding for classic windows

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ABSTRACT

It is well known that accurate frequency estimation of multi-frequency signals contaminated with additive noise is a common problem encountered in a large number of engineering practice scenarios. This paper proposes a new frequency estimator based on using zero-padding and main-lobe fitting techniques. The primary innovation of this new algorithm is that it can be applied to most classic window functions, including adjustable windows. Systematic errors for various windows are studied and algorithm stability with respect to white Gaussian noise is investigated. In addition, a comparative study demonstrates that the proposed algorithm is more robust against additive noise than traditional algorithms because of its insensitivity to the incorrect polarity estimation and an intrinsic ability for partially canceling noise influence. Featured with straightforward operation, sufficient accuracy, strong compatibility, as well as robustness towards additive noise, the proposed approach is an appropriate choice for frequency estimation in spectral analysis.

1. Introduction

Accurate parameters estimation (frequency, amplitude and phase) for sinusoids contaminated with random noise has been a subject of investigation in various fields for several decades. For example, a common problem encountered in vibration analysis for rotating machinery is parameter estimation of a sampled multi-frequency signals in the presence of additive noise [1]. Typically, signs of probable error occur if a new peak value emerges in the spectrum or the root-mean-square value of the vibration at an integer multiple of the fundamental frequency changes [1]. With the increasing application of non-linear devices and periodical time-variable loads in electrical power systems, distortion of current and voltage waveforms becomes a serious problem. Therefore, real-time analysis and control of electrical power harmonics is of great significance for maintaining electrical energy quality, preventing damage to electrical network systems, and saving energy [2,3]. Besides, a number of audio coding technologies have been recently developed, where the audio signal is decomposed into sinusoids and noise before coding. The decomposition, of course, depends on the accurate frequency estimation of the audio signal [4]. Prior literature introduced various estimation approaches which can be generally classified into the categories time domain and frequency domain. Given their straightforward operation and high efficiency, frequency domain approaches based on the discrete Fourier transform (DFT) and implemented by the fast Fourier transform (FFT) are often

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used. However, there are some inherent drawbacks to frequency domain methods, such as the picket fence effect (PFE) and the spectral leakage effect (SLE) [3]. They may introduce significant errors in the frequency estimates if the signal is non-coherently sampled [5,6]. Studies have shown that if one can afford additional computational cost, it is possible to compensate for errors and obtain highly accurate frequency estimates even with a small number of samples [1].

The interpolated DFT algorithm (IpDFT) is one of the most widely studied estimation methods [1–16]. The main idea of this method is that the frequency error can be compensated by the ratio of weighted average of a certain number of known spectral bins around the local maximal. When a maximum sidelobe decay window (MSDW, also known as Rife–Vincent class I windows) is selected, the frequency error can obtained by means of simple analytical relationships [11–14]. Based on the work of Offelli and Petri [10], Duda deduced the polynomial approximation interpolation algorithm (PAIpDFT) which can be used for Dolph–Chebyshev windows and Kaiser–Bessel windows [15]. Polynomial coefficients are required to be calculated before analysis, which means that coefficients for various windows must be calculated and saved in advance. This process is inevitably intensive and troublesome, particularly for adjustable windows whose properties can be regulated by one or more parameters.

In this paper, we first propose a new interpolation algorithm for the Hanning window in which the zero padding technique was adopted to obtain more spectral lines within a frequency interval. Subsequently, this new algorithm is extended for compatibility with other classic windows by introducing the main-lobe fitting technique. We studied systematic errors of the proposed method for various windows, as well as the performance under white Gaussian noise. Finally, a comparative study was done. We demonstrate by simulation that in the presence of noise the proposed algorithm was more robust than traditional algorithms.

2. Algorithm fundamentals

Let us consider, for simplicity but without loss of generality, the continuous cosine signal contaminated with additive white noise, which is given in the form

$$x(t)=A_0\cos(2\pi f_0 t+\theta_0)+e(t), \tag{1}$$

where A_0 denotes the amplitude, f_0 denotes frequency, θ_0 denotes phase angle, t denotes the continuous-time variable, and $e(t)$ is the white noise. Sampling at frequency f_s over the observation interval $N\Delta t(\Delta t=1/f_s)$, the following discrete cosine signal of N samples

$$x(n)=A_0\cos\left(2\pi\frac{f_0}{f_s}n+\theta_0\right)+e(n),\quad n=0,1\cdot\cdot\cdot,N-1, \tag{2}$$

becomes available. In order to satisfy the Ny-quist Sampling Theorem, f_s must be greater than $2f_0$. Frequency resolution is obtained by $\Delta f=f_s/N$. The ratio of theoretical frequency to sampling frequency is given by

$$\frac{f_0}{f_s}=\frac{\lambda_0}{N}, \tag{3}$$

where λ_0 denotes the number of cycles contained in the signal samples [14]. It should be stressed that λ_0 also represents the normalized frequency (frequency f_0 scaled by frequency resolution) expressed in DFT bins [14]. At this stage, we ignore the noise term and proceed to multiply the signal samples $x(n)$ by the data window values $w(n)$. The weighted samples are given by

$$x_w(n)=A_0\cos(2\pi\lambda_0 n/N+\theta_0)w(n). \tag{4}$$

The discrete Fourier transform (DFT) of the weighted signal can be computed by

$$X_w(k)=\sum_{n=0}^{N-1}x_w(n)e^{-j\frac{2\pi}{N}nk}. \tag{5}$$

Applying (5) to weighted samples in (4) yields the explicit form

$$X_w(k)=\frac{A_0}{2}e^{j\phi_0}W_N(k-\lambda_0)+\frac{A_0}{2}e^{-j\phi_0}W_N(k+\lambda_0), \tag{6}$$

where $W_N(k)$ denotes the discrete time Fourier transform (DTFT) of the weighting function. If the signal is asynchronously sampled, the normalized frequency λ_0 lies between two largest spectral lines. Therefore, λ_0 can be further written into two parts with the integer part of l_w and the fractional part δ_w $(-0.5<\delta_w\le 0.5)$, respectively.

$$\lambda_0=l_w+\delta_w. \tag{7}$$

The integer part l_w can be readily and correctly determined by means of a maximum search routine, as long as the SNR is above threshold (approximately −18 to −20 dB) [17]. Substituting (7) into (6) gives the largest magnitude

$$|X_w(l_w)|=\frac{A_0}{2}\left|e^{j\phi_0}W_N(-\delta_w)+e^{-j\phi_0}W_N(2l_w+\delta_w)\right|. \tag{8}$$