Mechanical Systems and Signal Processing 70-71 (2016) 1073–1084

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

# A double modal synthesis approach for brake squeal prediction

M. Monteil [a], S. Besset [a,b], J.-J. Sinou [a,b,c]

[a] Laboratoire de Tribologie et Dynamique des Systèmes, UMR CNRS 5513, École Centrale de Lyon, 36 avenue Guy de Collongue 69134 Écully Cedex, France
[b] Centre Lyonnais d'Acoustique, Université de Lyon, 69622 Lyon, France
[c] Institut Universitaire de France, 75005 Paris, France

ARTICLE INFO

*Article history:*
Received 26 February 2015
Received in revised form
19 June 2015
Accepted 20 July 2015
Available online 9 September 2015

*Keywords:*
Friction-induced vibration
Stability analysis
Reduction strategy
Squeal

ABSTRACT

This paper is devoted to propose a new efficient reduction method for predicting the stability analysis of a brake system subjected to friction-induced vibration. The finite element brake system under study is composed of a disc and a pad. The contact is modeled by introducing contact elements at the friction interface with the classical Coulomb law and a constant friction coefficient. It will be demonstrated that it is possible to build efficient reduced finite element models by developing a reduced model based on a Double Modal Synthesis (i.e. a classical modal reduction via Craig & Bampton plus a condensation at the frictional interface). Special attention is being conducted to validate the convergence of the reduced model especially on the approximation of the unstable modes with respect to real and imaginary parts. This complete numerical strategy based on Double Modal Synthesis allows us to perform relevance squeal prediction of unstable vibration modes. It is demonstrated that the numerical results via the Double Modal Synthesis are in good agreement with those of the classical Craig & Bampton method.


## 1. Introduction

Research for predicting squeal noise has been regularly performed for many years [1–3]. Despite great progress in the understanding and numerical simulation of brake squeal (i.e. methodology for the modeling of the variability [6,7], damping effects on squeal [4,5], statistical analysis of brake squeal noise [8], non-linear formulations and behaviors at the frictional interfaces [9–12], chaotic phenomenon [13] or acoustic emissions [14–16]), there is still considerable progress to provide in order to achieve efficient numerical approaches for squeal prediction. One of the most important drawbacks that needs to be investigated is the capability to reduce computational time and data storage.

One of the possible ways is to propose minimal or reduced finite element models that can reproduce the propensity of squeal noise. In this work, the focus is on an original numerical modal reduction in order to predict the stability analysis of large finite element models for brake systems. Indeed, it appears that one of the major limitations of classical reductions for squeal noise is associated with the size of interface matrices due to the explicit use of the interface degrees of freedom (i.e. the size of the reduced finite element model is strongly dependent on the number of the degrees of freedom at the frictional interface). So an interesting concept should be to develop an implicit reduction at the frictional interface in order to return

*E-mail addresses:* melodie.monteil@gmail.com (M. Monteil), sebastien.besset@ec-lyon.fr (S. Besset), jean-jacques.sinou@ec-lyon.fr (J.-J. Sinou).

http://dx.doi.org/10.1016/j.ymssp.2015.07.023
0888-3270/

accurate results on stability analysis by keeping a minimal number of generalized degrees of freedom at the interface. This reduction (at the frictional interface) in connection with conventional reductions (for each of the sub-structures) must be able to reproduce the overall behavior of the system for the prediction of the squeal propensity. In this work, we propose a new original strategy based on the Double Modal Synthesis [17,18]: the reduced finite element model is performed by using the combination of the Craig & Bampton reduction procedure [19] with an interface reduction that allows to reduce the size of the internal interface between substructures.

This paper is divided into four parts: firstly, the finite element model under study for brake squeal is presented with the modeling hypotheses. Secondly, the classical Craig & Bampton reduction is briefly discussed and the relevance of this reduction method for the stability analysis of the proposed finite element model is studied in details. The last part is devoted to the presentation of the original proposed strategy based on the Double Modal Synthesis. Finally, discussion of numerical results is proposed. A comparison with the classical Craig & Bampton method (without reduction at the frictional interface) is carried out with an evaluation of the computational performances.

## 2. The model of the simplified brake system

This section is firstly devoted to present the finite element model of the simplified brake system under study. Secondly, the classical stability analysis for the complete system that will served as a reference for the following parts of the paper is discussed.

### 2.1. Finite element model

The finite element model under study corresponds to a simplified brake system that is composed of two isotropic elastic structures: a circular disc and a pad [12,20]. The Structural Dynamics Toolbox (SdTools – Matlab Software) is used to build the finite element model of the two substructures as shown in Fig. 1. Concerning the boundary conditions, the pad is in-plane fixed and the inner surface of the disc is clamped. Details on the finite element model and material properties, inspired from [15], are listed in Tables 1 and 2, respectively.

The equations of motion of the brake system can be written as

$$\mathbf{M}\ddot{\mathbf{X}}+\mathbf{C}\dot{\mathbf{X}}+\mathbf{K}\mathbf{X}=\mathbf{F}+\mathbf{F_p}, \quad (1)$$

where **M**, **C**, **K** are the mass, damping and stiffness matrices, respectively. $\ddot{\mathbf{X}}$, $\dot{\mathbf{X}}$, $\mathbf{X}$ are acceleration, velocity and displacement vectors. $\mathbf{F_p}$ defines the vector of the pressure force. **F** is related to the vector of forces due to contact and friction occurring at the disc/pad interface especially at the contact nodes. The contact force is described by the following mathematical function:

$$F_c^i=k_L(z_p-z_d), \quad (2)$$

where $z_p$ and $z_d$ define, respectively, the displacements of contact nodes for the pad and the disc at the $i$th contact element. $k_L$ is the contact stiffness coefficient fixed at $3.1\times10^7$ N.m$^{-1}$. This value has been chosen to fit the first order of pad compression curves obtained from experimental tests given in [20]. This value is only valuable over a predefined pressure range. Moreover, a simplified Coulomb law is considered with a constant friction coefficient without any stick-slip motion. Then, the friction force $F_f^i$, located at the $i$th node is derived from the contact force $F_c^i$ at the friction interface in the tangential plan with

$$F_f^i=\mu F_c^i, \quad (3)$$

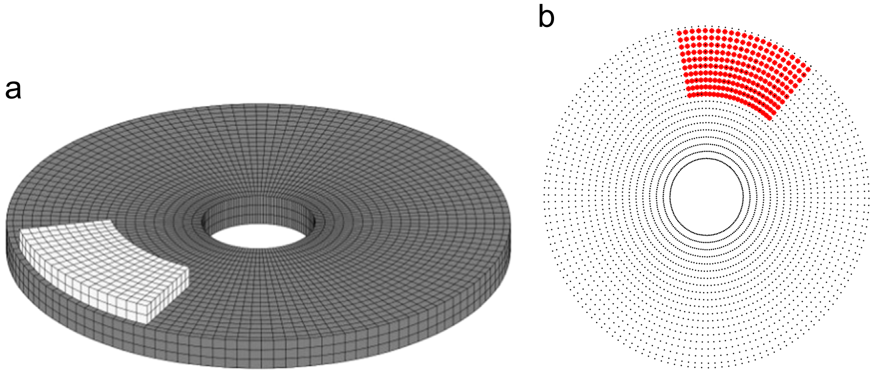


**Fig. 1.** Design of the finite element model. (a) Simplified brake system [15]. (b) 220 uniformly spaced contact nodes at the frictional pad/disc interface (in red colour). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)