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A comprehensive dynamic model to investigate the stability problems of the rotor–bearing system due to multiple excitations

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ABSTRACT

In this paper, a comprehensive dynamic model is proposed to analyze the dynamic behaviors of the rotor–bearing system. Three types of excitation including the bearing waviness, the unbalanced force and the finite number of balls (varying compliance effect) are considered. Based on the extended Jones–Harris model with five degrees of freedom, the load distribution and then the stiffness of the angular contact ball bearing are obtained theoretically. After introducing the three types of excitation into the model, the bearing stiffness matrix becomes time-varying and many parametric frequencies are found due to the multiple excitations. Then, the stability problems of the parametrically excited rotor–bearing system are investigated utilizing the discrete state transition matrix method (DSTM). The simple instability regions arising from the translational and the angular motions are analyzed respectively. The effects of the amplitude and the initial phase angle of the bearing waviness, the rotor eccentricity, the bearing preload and the damping of the rotating system on the instability regions are discussed thoroughly. It is shown that the waviness amplitudes have significant influences on the instability regions, while the initial phase angles of the waviness almost have no effect on the instability regions. The rotor eccentricity just affects the widths of the instability regions. The increasing of the bearing preload only shifts the positions of the instability regions. Damping could reduce the instability regions but it could not diminish the regions completely.

1. Introduction

Rolling element bearings are among the most important components in rotating machinery. Up to now, vibration and noise are still the most common problems in the bearing applications [1]. As a result of the imperfect manufacturing process and the inherent properties of the bearing, these problems are usually inevitable. Therefore, to predict the working conditions leading to such problems and to find the effective measures to attenuate or avoid their adverse effects are especially important. It is just the purpose for the stability analysis of the rotating system.

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Nomenclature

c_{xx}, c_{yy}, c_{zz}, $c_{\varphi_x\varphi_x}$, $c_{\varphi_y\varphi_y}$ damping coefficients of the rotor–bearing system
d_m diameter of pitch circle of the bearing
D_m, nominal diameter of ball
D_{ij}, D_{oj} diameter of ball contacting with inner race and outer race, respectively
e eccentricity of rotor
F_u amplitude of unbalanced force
F_{cj} centrifugal force of ball
F_x, F_y, F_z static forces on the bearing
$\overline{F}$ unbalanced force
$\overline{F}_x$, $\overline{F}_y$, $\overline{F}_z$ time-varying forces on the bearing
f_{ec}, f_{VC}, f_{wb}, f_{wi}, f_{wo} parametric frequencies due to imbalance, finite number of balls, ball waviness, inner race waviness, outer race waviness, respectively
g gravitational acceleration
I_d, I_p diametral moment of inertia and polar moment of inertia
j sequence number of balls
k_{xx}, k_{yy}, bearing stiffness coefficients
k'_{xx}, k'_{yy}, k'_{zz}, $k'_{\varphi_x\varphi_x}$, $k'_{\varphi_y\varphi_y}$ stiffness coefficients of the rotor–bearing system
K Hertzian contact coefficient
L half of the length of the shaft
l waviness order
l_{wb}, l_{wi}, l_{wo} order of ball waviness, inner race waviness and outer race waviness, respectively
M_{gj} gyroscopic moment of the jth ball
M_x static moment on the bearing
$\overline{M}_x$, $\overline{M}_y$, time-varying moments on the bearing
m mass of the disk
P_{il}, P_{ol}, amplitude of inner and outer race waviness with order l
p_{ij}, p_{oj} inner and outer race waviness contacting with the jth ball
q positive integer
R radius of pitch circle
R_i radius of locus of inner race groove curvature centers
r_{ij}, r_{oj} groove curvature radii of inner and outer races contacting with the jth ball
r_{im}, r_{om} nominal groove curvature radii of inner and outer races, respectively
t time
U_{jl} amplitude of waviness with order l on the jth ball
u_{ij}, u_{oj} waviness on the jth ball contacting with the inner and outer races, respectively
X_{1j}, X_{2j} auxiliary variables for the geometrical relationship at the jth ball
x, y, z transverse and longitudinal displacements of the disk
Z number of balls in the bearing
α nominal contact angle of the bearing
δ_{ij}, δ_{oj} ball deformation between ball-inner race and ball-outer race, respectively
δ_x, δ_y, δ_z translational displacements of the inner race
θ_x, θ_y, angular displacements of the inner race
λ_i the ith characteristic multiplier of the discrete state transition matrix
λ_{ij}, λ_{oj} friction constant at the jth ball
η_{jl} initial phase angle of waviness with order l on the jth ball
ξ_{il}, ξ_{ol} initial phase angle of inner and outer race waviness contacting with the jth ball
φ_x, φ_y angular displacements of the disk
ψ_j position angle of the jth ball
Ω rotating frequency of the shaft
Ω_p central position of the instability region indicated by frequency
ω_b, ω_c, ω_i, ω_o, angular speeds of ball, cage, inner race and outer race, respectively
$\overline{\omega}_i$, $\overline{\omega}_j$ parametric frequency

The waviness, the unbalanced force and the finite number of balls are important excitations of severe vibrations in the rolling element bearing. Waviness is the global sinusoidally shaped imperfection on the surfaces of the bearing components. In general, these imperfections are due to the irregularities during the grinding and honing process of the bearing components [2]. Nowadays, the amplitude of the waviness in rolling element bearings is at the micrometer level. Despite that, waviness can still produce significant vibrations. In this field, one of the first systematic investigations was made by Tallian and Gustafson [3]. In the research, it was pointed out that race waviness with lower order affected the amplitude of the vibrations at the ball passage frequency. From the point of view of bearing manufacturing, Yhland [4] firstly focused on the experimental measurement of waviness. Subsequently, Yhland [5] proposed a linear analytical model to investigate the effect of geometrical imperfections on the vibration of the rotor–bearing assembly. However, the centrifugal forces of the bearing were neglected in his analysis. Theoretically and numerically, Wardle [6,7] summarized the relations between the vibration frequencies and the wave number of the surface waviness. It was pointed out that only waviness with special orders would cause noticeable vibrations. Aktürk [8] investigated the effect of bearing waviness on the vibration of the shaft. The research revealed that the relationship between waviness and vibration spectrum was very complicated. Jang and Jeong [9–11] carried out a series of work to analyze the effects of bearing waviness on the vibration forces and frequencies and on the stability of the rotating system. Based on a five-degree-of-freedom model, they characterized the vibration frequencies due to the various types of waviness. Although the stability issues due to the waviness excitation were analyzed systematically, the effects of waviness parameters and loading conditions of the bearing on the instability regions were not considered in their study. Sopanen and Mikkola [12,13] proposed a detailed dynamic model of the deep-groove ball bearing taking various factors into consideration. Their model improved the dynamic analysis of the rotor–bearing system. Yet, the