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# Introducing angularly periodic disturbances in dynamic models of rotating systems under non-stationary conditions

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ABSTRACT

A mechanical defect in a rotating system can be interpreted as the presence of a periodic angular perturbation. Understanding the interactions between the defect and its consequences on measurable quantities representative of dynamic behavior (acceleration, speed, etc.) necessarily involves the implementation of numerical models. The aim of this paper is to present a model suitable for introducing this kind of disturbance in the case of systems operating under non-stationary speed conditions. This model corresponds to a numerical extension of approaches increasingly used to analyze experimental signals.

The resulting model can be used to investigate of the influence of disturbance parameters on the dynamic responses of the rotating system. It also allows comparisons of new assumptions on the origin of this perturbation. This new approach is illustrated by using the general framework of bearing faults.



## 1. Introduction

### 1.1. General background

This work is part of the broader framework of monitoring and maintaining rotating machinery and more specifically diagnosing localized faults in component with periodic geometry, such as gears and bearings. The experimental work carried out on many devices and various operating conditions have shown that the presence of a localized defect on a gear tooth or a bearing ring will lead to small changes in the dynamic behavior of the device. For defects of small dimension resulting disturbances are of short time of duration but appear cyclically with an angular period which depends only on the geometry of the defective component. Most diagnostic techniques are based on these characteristic frequencies to detect the presence of a fault. For gears it is the meshing frequency. It only depends on the number of teeth on gears. The bearings also have characteristic frequencies which depend on their geometry such as the rolling elements pass frequency on the outer ring (BPFO). The calculation of these frequencies requires knowing precisely the internal geometry of the bearing which is rarely the case. But most bearing manufacturers provide these characteristic frequencies as well as load capacities. However, it should be noted that these frequencies are calculated for ideal operating conditions and without slip and they can vary from 1% to 2% in reality. The challenge for monitoring this kind of components is to develop tools able to detect these small variations of the dynamic behavior and then analyzed them to obtain information about the fault. The vast majority of these works concern the study of accelerometer signals in which the perturbations due to the presence of a fault can be detected.

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The work presented in this paper involves the analysis of Instantaneous Angular Speed (IAS) signals and the use of angular sampling. Angular sampling can be achieved directly or after time sampling with different interpolation procedures [1]. All these angular approaches are now very well-known [2] and used for different kinds of signals, classically for accelerometer signals. Several authors have demonstrated the effectiveness of IAS signals [3] and used them in various applications [4,5,6]. IAS is very useful and sensitive for monitoring [7] and diagnosing different faults in rotating components with periodic geometry under variable operating conditions [6,8]. Nevertheless, these experimental approaches do not enable understanding how a fault on the outer ring can lead to rotating speed variations. They must be coupled with the formulation of numerical models in which the operating conditions and the geometry of the device can be controlled and changed. Building such models requires the introduction of perturbations that are supposed to be representative of the defects in the mechanism. In this context, it seems natural to extend the angular approach implemented experimentally in numerical models [9]. Since the approach proposed in this paper is very new and different from classical modeling, it has been difficult to find similar modeling methods in other scientific fields and, to the knowledge of the authors, the literature on the subject appears sparse.

### 1.2. General model

We consider the case of a rotating flexible shaft modeled by a discrete representation with n degrees of freedom (DOF), which can be expressed by the following matrix differential equation:

$$[\mathrm{M}]_{n\times n}\cdot\{\ddot{\mathrm{X}}\}_n+[\mathrm{C}]_{n\times n}\cdot\{\dot{\mathrm{X}}\}_n+[\mathrm{K}]_{n\times n}\cdot\{\mathrm{X}\}_n=\{\mathrm{F}_{ext}\}_n \tag{1}$$

where {X} is the vector of generalized displacements, [M], [C], [K] are the matrices of mass, damping, stiffness and $\{\mathrm{F}_{ext}\}$ the vector of external forces (driving and resistant forces). The system defined by Eq. (1) has a rigid mode corresponding to the free rotation of the shaft.

This system of $n$ differential equations of order 2 can be rewritten as a system of $2n$ equations of order 1 by setting:

$$\{\mathrm{Q}\}_{2n}=\begin{Bmatrix}\{\mathrm{X}\}_n\\ \{\dot{\mathrm{X}}\}_n\end{Bmatrix} \tag{2}$$

Then, system (1) becomes:

$$\begin{bmatrix}[\mathrm{I}_d] & [0]\\ [0] & [\mathrm{M}]\end{bmatrix}\cdot\begin{Bmatrix}\{\dot{\mathrm{X}}\}\\ \{\ddot{\mathrm{X}}\}\end{Bmatrix}=\begin{Bmatrix}\{0\}\\ \{\mathrm{F}_{ext}\}\end{Bmatrix}-\begin{bmatrix}[0] & -[\mathrm{I}_d]\\ [\mathrm{K}] & [\mathrm{C}]\end{bmatrix}\cdot\begin{Bmatrix}\{\mathrm{X}\}\\ \{\dot{\mathrm{X}}\}\end{Bmatrix} \tag{3}$$

where $[\mathrm{I}_d]$ is the identity matrix of dimension $n$. Thus if the derivative of the state vector can be expressed as

$$\{\dot{\mathrm{Q}}\}=\begin{bmatrix}[\mathrm{I}_d] & [0]\\ [0] & [\mathrm{M}]^{-1}\end{bmatrix}\cdot\left(\begin{Bmatrix}\{0\}\\ \{\mathrm{F}_{ext}\}\end{Bmatrix}-\begin{bmatrix}[0] & -[\mathrm{I}_d]\\ [\mathrm{K}] & [\mathrm{C}]\end{bmatrix}\cdot\{\mathrm{Q}\}\right) \tag{4}$$

then Eq. (1) can be written as

$$\{\dot{\mathrm{Q}}\}=\begin{bmatrix}[\mathrm{I}_d] & [0]\\ [0] & [\mathrm{M}]^{-1}\end{bmatrix}\cdot\begin{bmatrix}[0] & [\mathrm{I}_d]\\ -[\mathrm{K}] & -[\mathrm{C}]\end{bmatrix}\cdot\{\mathrm{Q}\}+\begin{bmatrix}[\mathrm{I}_d] & [0]\\ [0] & [\mathrm{M}]^{-1}\end{bmatrix}\begin{Bmatrix}\{0\}\\ \{\mathrm{F}_{ext}\}\end{Bmatrix} \tag{5}$$

## 2. Parametric modeling of disturbance and related issues

### 2.1. Definition of disturbance

The disturbance considered in this document must be representative of the presence of a fault on a moving mechanical element of a rotating system, so it will be angularly periodic. Let $\theta$ be the angular degree of freedom associated with the mechanical component having the defect. Thereafter $\theta$ is called "master dof". The presence of this defect will change the dynamic behavior of the device and consequently will introduce angular speed variations. These variations are very small and must be distinguished from macroscopic velocity variations representative of non-stationary operating conditions.

From the mechanical point of view, this disturbance can be equivalent to a variation of the external forces $\{\Delta F_p(\theta)\}$ and/or, for the defective component, to a variation of its stiffness $[\Delta K_p(\theta)]$ and/or to a variation of its damping $[\Delta C_p(\theta)]$. Current knowledge does not allow choosing for one of these assumptions. The most suitable modeling will be defined in future works by using comparisons between experimental and simulation results. In the general case where the presence of a defect modifies the external forces, damping and rigidity, dynamic Eq. (1) governing the behavior of the system becomes:

$$[\mathrm{M}].\{\ddot{\mathrm{X}}\}+[[\mathrm{C}]+[\Delta C_P(\theta)]]\cdot\{\dot{\mathrm{X}}\}+[[\mathrm{K}]+[\Delta K_P(\theta)]]\cdot\{\mathrm{X}\}=\{\mathrm{F}_{ext}\}+\{\Delta\mathrm{F}_p(\theta)\} \tag{6}$$