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The velocity synchronous discrete Fourier transform for order tracking in the field of rotating machinery

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ABSTRACT

Diagnostics of rotating machinery has developed significantly in the last decades, and industrial applications are spreading in different sectors. Most applications are characterized by varying velocities of the shaft and in many cases transients are the most critical to monitor. In these variable speed conditions, fault symptoms are clearer in the angular/order domains than in the common time/frequency ones. In the past, this issue was often solved by synchronously sampling data by means of phase locked circuits governing the acquisition; however, thanks to the spread of cheap and powerful microprocessors, this procedure is nowadays rarer; sampling is usually performed at constant time intervals, and the conversion to the order domain is made by means of digital signal processing techniques. In the last decades different algorithms have been proposed for the extraction of an order spectrum from a signal sampled asynchronously with respect to the shaft rotational velocity; many of them (the so called computed order tracking family) use interpolation techniques to resample the signal at constant angular increments, followed by a common discrete Fourier transform to shift from the angular domain to the order domain. A less exploited family of techniques shifts directly from the time domain to the order spectrum, by means of modified Fourier transforms. This paper proposes a new transform, named velocity synchronous discrete Fourier transform, which takes advantage of the instantaneous velocity to improve the quality of its result, reaching performances that can challenge the computed order tracking.

1. Introduction

Most signals measured in the field of machine condition monitoring present strong, often dominant, harmonic components, synchronous with the shaft rotation. For instance, the analysis of signal for the diagnostics of rotors [1] and gears [2] is usually based on integer multiples of the shaft velocity. Considering that also the cyclostationary components, the second biggest class in machine diagnostics, present cyclic frequencies strictly related to the shaft velocity [3], it is clear that the monitoring of rotating machineries is by far more effective if analyses are performed in the *angular domain* of the shaft rotation, rather than in *time domain*. The same conclusion is valid for the order domain, the spectral counterpart of the angular domain (measured in *n per revolution*, *n*X), which is always preferred to the "Hertzian" frequency domain.

The idea of synchronously sampling mechanical signals came from the field of electronics and telecommunications, where, a wide and detailed literature was available starting from the '60s, see for instance [4,5] and [6]. Prior to the diffusion of cheap micro-processors the most feasible option consisted in electronic circuits able to generate triggering waves for the

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acquisition system, phase-locked with a speed sensor installed on the machine [7]. Thanks to these hardware systems, the sampling of the signal could be performed directly in the angular domain, i.e. at constant shaft angular increments, instead of constant time intervals.

The availability of cheap digital signal processors has made a second option more economically appealing to obtain signals in the angular domain. It consists in digitally processing the signal in order to obtain the domain transformation. In this way, the complexity of the hardware is significantly reduced, and the system becomes much more flexible for modifications. The first attempts in this direction started in the late 80s [8], to reach the most significant results in the 90s, with the works of Fyfe and Munck [9], Stander and Heyns [10], Vold and Leuridan [11], Blough, Brown and Vold [12]. The different approaches may be grouped in three big families of order tracking techniques: the *resampling methods*, the *Kalman filter based methods* and the *transform based methods*.

The resampling methods, or *computed order tracking* (COT), are the most intuitive approach: they work mainly in time domain, interpolating the signal to obtain a result which should represent a signal sampled by a hardware-synchronized acquisition system. The second family is based on non-casual filtering operations [16], which, in some implementations, can be recursively adjusted [17]. The third family of order tracking techniques is based on integral transforms, able to extract, with some degree of accuracy, the order domain information directly from the time-domain signal. Such transforms are generally derived from the Fourier transform, with proper modifications of the kernel [12]. This paper focuses on this last family of order tracking techniques, which present the advantage of a direct passage from the signal sampled uniformly in time domain to the order spectrum. In particular the paper introduces a new transform, the *velocity synchronous discrete Fourier transform* (VSDFT), exploiting the estimate of the instantaneous angular speed (IAS) to perform the order tracking. The estimation of the instantaneous angular speed [13–15] is not the focus of this paper, which deals with its use for the aforementioned purpose of order tracking.

2. Order tracking: a multi-domain problem

The *order spectrum*, the angular counterpart of the common frequency spectrum, is obtained by Fourier transforming the signal from the angular domain:

$$X(\Omega) = \int_{-\infty}^{+\infty} x(\theta) e^{-j\Omega\theta} d\theta \tag{1}$$

To practically implement this operation directly by means of *discrete Fourier transform* (DFT), a (synchronous) constant angular sampling should be implemented. However acquisition systems are seldom programmed to acquire data in a synchronous way and signal are usually sampled at constant time intervals, irrespective of the actual instantaneous velocity of the shaft.

Among the different techniques developed in literature to overcome this problem, two are worth mentioning to introduce the technique proposed in this paper. The aforementioned COT focuses on the modification of the signal $x(t)$ itself, transforming it to the angular domain by means of numerical interpolations. The resulting signal $x(\theta)$ is sampled at constant angular increments and can directly undergo the digitalized form of the transformation of Eq. (1), consisting in practice of a common DFT in the new angular domain:

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n\Delta\theta] e^{-j\Omega[k] n\Delta\theta} \tag{2}$$

where $\Delta\theta$ and N are respectively the angular resolution and the number of samples of the interpolated signal $x(\theta)$, and $\Omega[k]$ the vector of orders for the representation of the order spectrum. The second technique, belonging to the family of integral transforms for order tracking and proposed by Blough et al. [12], exploits the possibility of changing the kernel of the transformation itself, to pass directly from the time domain to the order domain $\{\Omega\}$. Such DSP technique, a particular case of the chirp-z transform, is called *time variant Fourier transform* (TVDFT), whose expression is the following:

$$\text{TVDFT}[k] = \frac{1}{N} \sum_{n=1}^{N} x[n\Delta t] e^{-j\Omega[k] \int_0^{n\Delta t} \omega[n] dt} \tag{3}$$

The modified kernel, characterized by the instantaneous frequency of the tachometer ω, allows processing the signal $x[n\Delta t]$ without prior resampling. However, such transform shows limits for highly variable shaft speeds and is characterized by a set of non-orthogonal kernels. Blough proposed the introduction of an *orthogonality compensation matrix* (OCM) to partially correct this problem [18].

Significant improvements can be obtained reconsidering the theoretical basis of the domain transformation, restarting from Eq. (1), at the basis of COT, and operating a change of variable in the domain of integration:

$$X(\Omega) = \int_{-\infty}^{+\infty} x(\theta(t)) e^{-j\Omega\theta} \frac{d\theta}{dt} dt \tag{4}$$