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Using the moving synchronous average to analyze fuzzy cyclostationary signals

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ABSTRACT

Cyclostationarity is a property of vibration and acoustic signals recorded on rotating machines operating at constant speed. It states that the statistic properties of signals are periodic: the random process defined by the signal observed at a given position in the cycle is stationary, the cycle being defined as the angle interval between two identical configurations of the mechanical system. Cyclostationarity is not fully satisfied if the signals are acquired in the time domain on rotating machines with a fluctuating rotation speed. Indeed, if the instantaneous rotation speed is not purely periodic, it means that time samples taken at a constant time interval (equal to the average cycle duration) do not correspond exactly to an angle in the cycle. In this particular case, a synchronous averaging of cycle realizations can still be processed to estimate a periodic part using a predefined trigger angle to align cycle realizations before the averaging process. In these conditions, the synchronous average depends on the chosen synchronization angle: each point of the synchronous average is an estimate of the expected value of the signal at a given time preceding or following the synchronization angle. The synchronous average can be computed as a function of the synchronization angle, varying over an entire cycle. The result is a moving synchronous average that can be post-processed for diagnosis purposes. For example, a time frequency representation of the moving synchronous average can be computed, and the synchronization angle maximizing each point of the time frequency map can be easily extracted. Under certain conditions of instantaneous speed fluctuations, this analysis allows the precise localization of different mechanical events in the cycle, as well as their contributions in the analyzed vibration or acoustic signal. The analysis of the moving synchronous average leads also to the estimation of the energy loss of the synchronous average processed in angle caused by cycle-to-cycle speed fluctuations.

1. Introduction

The synchronous average is a signal processing tool consisting in averaging several portions of signal chosen in function of the studied mechanical system [1]. For cyclostationary signals, the synchronous average is an estimation of the deterministic or periodic part [2]. It can also be regarded through comb filtering [3], the signal is filtered by a specific comb resulting in an averaging process over signal samples separated by a specific delay. Noise and vibration signals recorded on rotating machines are determined by cyclic excitations defined in angle, and the response of the structure is defined by its

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Nomenclature

t	time history
θ	(unwrapped) rotation angle
Θ	angular cycle length
$\widehat{\Omega}$	average rotation speed
$\Omega(\theta)$	Instantaneous Angular Speed (IAS) as a function of θ
$f(\theta)$	elapsed time as a function of θ
$\overline{\Omega}(\theta)$	IAS of the mean cycle ($\theta \in [0 \ldots \Theta[$)
$\sigma_v(\theta)$	normalized cycle-to-cycle fluctuation of the IAS ($\theta \in [0 \ldots \Theta[$)
$\bar{f}(\theta)$	elapsed time as a function of θ for the mean cycle ($\theta \in [0 \ldots \Theta[$)
$\overline{T}$	duration of the mean cycle
ADP(θ)	Angular Deterministic Part $\theta \in [0 \ldots \Theta[$
TDP$_\theta(t)$	Temporal Deterministic Part ($t \in [-\infty \ldots \infty[$) for the synchronization angle $\theta \in [0 \ldots \Theta[$
ASA(θ)	Angular Synchronous Average ($\theta \in [0 \ldots \Theta[$)
TSA$_\theta(t)$	Temporal Synchronous Average, $t \in [-\overline{T}/2 \ldots \overline{T}/2[$ for the synchronization angle $\theta \in [0 \ldots \Theta[$
MTSA(θ,t)	Moving TSA, $\theta \in [0 \ldots \Theta[$, $t \in [-\overline{T}/2 \ldots \overline{T}/2[$
$s_{rms}(t)$	RMS value over a short sliding time window of $s(t)$
$s_{tfm}(t,f)$	time frequency distribution of $s(t)$

time domain impulse response, the determinism of such signals being consequently dual [4]. If the studied rotating machine is operating with a strictly constant or purely periodic IAS (Instantaneous Angular Speed), the time and angular clocks are totally dependent, and this duality is masked; and processing signals in time or angle will lead to equivalent results. If not, the duality leads to the so-called fuzzy-cyclostationary signals [5], and the result of synchronous averaging will be different in time or angle. The fuzzy-cyclostationary concept is established in the first section of this paper. The time domain synchronous average is redefined in this context in the second part of this work. It requires especially to define a synchronization angle, and its computation for several synchronization angles in the cycle is defined here as the moving synchronous average. The concepts proposed in this study are illustrated by two experimental examples presented in the third section. The fourth part shows how ASA and TSA (Angular and Temporal Synchronous Averages) can be compared. Then, the moving synchronous average is introduced, and different post-processing tools are proposed to extract some information about the studied rotating machine.

2. A definition of fuzzy cyclostationarity

The concept of fuzzy cyclostationarity is deeply inspected in Ref. [5] (in French). A brief definition is however given here, in order to clearly establish the general framework of this study.

Let $\omega(t)$ represent the instantaneous angular speed (IAS, rad/s) of a rotating machine as a function of time, and f the bijection between the temporal and angular domains such as $t = f(\theta)$. The instantaneous angular speed can be expressed as a function of the angle:

$$\Omega(\theta) = \omega(f(\theta)). \tag{1}$$

In stationary operating conditions, $\Omega(\theta)$ is assumed to be cyclostationary of cycle length Θ. The bijection f can be expressed as a function of Ω:

$$f(\theta) = t = \int_0^t d\tau = \int_0^\theta \frac{d\alpha}{\Omega(\alpha)}. \tag{2}$$

The statistical properties of $\Omega(\theta)$ are Θ-periodic. The angle to time relationship $f(\theta)$ can be written at $\theta+\Theta$ as follows:

$$f(\theta+\Theta) = \int_0^\theta \frac{d\alpha}{\Omega(\alpha)} + \int_\theta^{\theta+\Theta} \frac{d\alpha}{\Omega(\alpha)} = t + T(\theta). \tag{3}$$

The IAS at $\theta+\Theta$ is thus equal to

$$\Omega(\theta+\Theta) = \omega(f(\theta+\Theta)) = \omega(t + T(\theta)). \tag{4}$$

If $T(\theta) = T$ is constant, it means that the cyclostationarity of Ω in angle implies the cyclostationarity of ω in time. According to Eq. (3), this is the case if $\Omega(\theta)$ is constant or periodic. If the cyclic variance of $\Omega(\theta)$ is not null, then $T(\theta)$ depends on θ and some fluctuations of the cycle length will appear in time. In this case, $\omega(t)$ keeps the property of cyclostationarity, but with a cycle length equal to the average value of $T(\theta)$. It means that cycle realizations in the time domain will not correspond exactly to a cycle length Θ in angle: that is why this kind of cyclostationary process is qualified as fuzzy.

An important consequence of fuzzy cyclostationarity is that cyclic estimators, like the cyclic average or variance, will lead to different results if processed using a constant time or angle sampling. The present work aims to study these differences, and to present how the fuzzy cyclostationarity property can be used for diagnosis purposes.