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Time–frequency analysis for parametric and non-parametric identification of nonlinear dynamical systems

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ABSTRACT

This paper points out the differences between linear and nonlinear system identification tasks, shows that time–frequency analysis is most appropriate for nonlinearity identification, and presents advanced signal processing techniques that combine time–frequency decomposition and perturbation methods for parametric and non-parametric identification of nonlinear dynamical systems. Hilbert–Huang transform (HHT) is a recent data-driven adaptive time–frequency analysis technique that combines the use of empirical mode decomposition (EMD) and Hilbert transform (HT). Because EMD does not use predetermined basis functions and function orthogonality for component extraction, HHT provides more concise component decomposition and more accurate time–frequency analysis than the short-time Fourier transform and wavelet transform for extraction of system characteristics and nonlinearities. However, HHT's accuracy seriously suffers from the end effect caused by the discontinuity-induced Gibbs' phenomenon. Moreover, because HHT requires a long set of data obtained by high-frequency sampling, it is not appropriate for online frequency tracking. This paper presents a conjugate-pair decomposition (CPD) method that requires only a few recent data points sampled at a low-frequency for sliding-window point-by-point adaptive time–frequency analysis and can be used for online frequency tracking. To improve adaptive time–frequency analysis, a methodology is developed by combining EMD and CPD for noise filtering in the time domain, reducing the end effect, and dissolving other mathematical and numerical problems in time–frequency analysis. For parametric identification of a nonlinear system, the methodology processes one steady-state response and/or one free damped transient response and uses amplitude-dependent dynamic characteristics derived from perturbation analysis to determine the type and order of nonlinearity and system parameters. For non-parametric identification, the methodology uses the maximum displacement states to determine the displacement–stiffness curve and the maximum velocity states to determine the velocity–damping curve. Numerical simulations and experimental verifications of several nonlinear discrete and continuous systems show that the proposed methodology can provide accurate parametric and non-parametric identifications of different nonlinear dynamical systems.

1. Introduction

Mechanical systems are often built with unwanted or wanted nonlinearities [1–10]. Large elastic deformations introduce geometric nonlinearity, deformation-dependent material properties result in material nonlinearity, and other

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nonlinearities can be caused by backlash, clearances between mounting brackets, geometric constraints on deformation, misalignment of substructures, dry friction, and many types of nonlinear hysteretic damping, including aerodynamic damping, damping of shock absorbers for vehicles, and material damping of shape memory alloys and other materials [1–6]. For example, highly flexible deployable/inflatable structures are often used in space structural systems, advanced vehicles, many mechanical and civil systems, and medical devices and implants in order to reduce weight/space, increase speed/efficiency, and/or provide special mechanisms [2,4,6]. Such structures are built with wanted geometric nonlinearities.

Damage introduces extra nonlinearities, and aging aggravates nonlinearities. Hence, dynamics of actual mechanical systems is often nonlinear [7–10]. For example, dry friction between fractured surfaces is a nonlinear effect, and the breathing of a crack causes nonlinear intermittent transient response. Also, demountable/retractable civil structures are built with nonlinearities caused by loose joints, and the usage/aging aggravates such nonlinearities. Because parametric models of such nonlinearities are often difficult to determine, non-parametric system identification is more feasible for such systems.

Nonlinearity identification and damage inspection are very challenging reverse engineering tasks. For several decades structural engineers have been developing dynamics-based methods for fast and reliable system identification and damage inspection of large structures [7–10]. For reverse engineering, it is important to design experiments to limit the number of system parameters involved in each set of experimental data in order to increase the possibility of unique identification results. But, it is even more important to have a signal processing and data mining method that can extract from each set of experimental data as many system parameters as possible in order to reduce the time and cost of experiments. Hence, signal processing plays the key role in nonlinearity identification, but the challenge is how to obtain accurate extraction/tracking of dynamic characteristics (e.g., natural frequencies, mode shapes, mass, damping, stiffness, and external loading) from nonlinear noise-contaminated signals.

Most signal processing methods for dynamics characterization require frequency- and/or time-domain signal processing techniques. Transforming time-domain data into the frequency domain introduces errors and numerical noise, and some methods even require the frequency domain data to be transformed back into the time domain, which introduces even more errors. For transformation from the time domain to the frequency domain, the first step is to choose a complete set of basis functions with each one having a pre-determined frequency (e.g., Fourier transform) or a narrow frequency band (e.g., wavelet transform), and the second step is to perform convolution computation using the chosen basis functions to extract components similar to the basis functions. Unfortunately, this approach is not suitable for a nonlinear and/or nonstationary signal because its frequency changes with time. Dynamics with a changing frequency is sometimes called high dynamics. For high dynamics, adaptive basis functions need to be used, and they can only be derived from the signal itself. Current literature review indicates that Hilbert–Huang transform (HHT) is better than short-time Fourier transform, Wigner–Ville distribution, and wavelet transform for time–frequency analysis of nonlinear nonstationary signals because HHT does not use predetermined basis functions and function conformality for component extraction [11,12]. Moreover, HHT provides concise decomposition of a signal into just a few intrinsic mode functions (IMFs) by using the empirical mode decomposition (EMD), and each extracted IMF is often physically meaningful [11,12]. Furthermore, HHT provides more accurate time-varying amplitudes and frequencies of each IMF by using Hilbert transform (HT). Unfortunately, the accuracy and capability of HHT suffers from the end effect and several mathematical and numerical problems [13]. Moreover, how to identify and estimate nonlinearities from time-varying amplitudes and frequencies is an important but challenging task [14]. Hence, new developments in time–frequency analysis and nonlinearity identification are needed.

Because dynamic characteristics of nonlinear systems change with time, this work shows that time–frequency analysis is most appropriate for their characterization and identification. Moreover, this work develops and experimentally verifies time–frequency signal processing methods and identification techniques for parametric and non-parametric identification of nonlinear dynamical systems. We show that the type, order, and magnitude of nonlinearity of a nonlinear system can be determined by performing time–frequency analysis of just a few transient responses and/or steady-state responses to harmonic excitations.

2. Identification of nonlinear systems

Nonlinear systems have dynamic characteristics very different from those of linear systems in many aspects and in time and frequency domains, and hence approaches for their identification are different. For example, the steady-state response of a linear system to a harmonic excitation is not affected by initial conditions and it vibrates at the excitation frequency with a constant amplitude. Hence, the start-up transient part due to non-zero initial conditions can be well separated from the steady-state part, which enables the use of just one forced transient response for time-domain system identification [15]. On the other hand, a harmonically excited response of a nonlinear system cannot be decomposed into a constant amplitude steady-state part and a transient part only due to non-zero initial conditions. Hence, for a nonlinear system, a transient response and a steady-state response need to be separately obtained in order to perform accurate system identification, as shown later in Section 4.

For a nonlinear system, because the principle of superposition is invalid and its steady-state response frequency can be different from the excitation frequency, system identification is difficult to be performed in the frequency domain. Next we