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The parametric characteristic of bispectrum for nonlinear systems subjected to Gaussian input

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ABSTRACT

The bispectrum defined in terms of the third-order cumulant or moment is fairly sensitive to the non-Gaussianity of signals and, therefore, can effectively extract information due to deviations from Gaussianity. This property enables the bispectrum to detect and characterize the nonlinearity of a system as the output response of a nonlinear system subjected to a Gaussian input will no longer be Gaussian. In this study, for the polynomial nonlinear systems which can be modeled as a convergent Volterra series, an analytical expression about the calculation of the bispectrum for nonlinear systems subjected to zero mean Gaussian excitation was derived. Moreover, based on the expression, an explicit relationship was established as a polynomial function between the nonlinear characteristic parameters and the bispectrum, which greatly facilitates the analysis of how the former affects the latter. Numerical examples were included to verify the results.

1. Introduction

Over the past decades, high order spectra (HOS) [1], also called as polyspectra, have established status as a sophisticated mathematical and signal processing tool for nonlinear system analysis. It is known that the traditional power spectrum, which is formally defined as the Fourier transform (FT) of the autocorrelation sequence (the second-order cumulant), does not carry any information about the phase of system frequency response function (FRF) and is also unable to give any evidence of nonlinearity for a system. However, the HOS [2] is defined as the multidimensional Fourier transform of higher order cumulant of stationary random process and so can overcome the incompetent of power spectra.

When the third order cumulant is concerned, the corresponding HOS is called as the bispectrum, which is most familiar to people and has received most attention in literature. The bispectrum [3] is fairly sensitive to the non-Gaussianity of signals and can effectively extract information due to deviations from Gaussianity. If a signal is Gaussian, then its bispectrum would be identically zero; however, for a non-Gaussian signal, the bispectrum can be non-zero, depending on whether the signal is symmetrically distributed or not. It was further revealed that, for linear systems, if the system input is a Gaussian process, then the system output will be Gaussian as well and consequently the output bispectrum is zero. On the other hand, for nonlinear systems, a Gaussian input will not definitely generate a Gaussian output and so the output bispectrum could be non-zero. More specifically, if the system nonlinearity is non-symmetrical, then the output bispectrum is non-zero. This property allows us to design a simple hypothesis test to detect the nonlinearity existence.

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Nomenclature

m, c, k	Mass, damping and stiffness coefficients, respectively
k_2, k_3	Quadratic and cubic nonlinear stiffness parameters, respectively
x, y	System input and output, respectively
$X(.)$, $Y(.)$	Fourier spectra of system input and output, respectively
$C_{x,2}$, $C_{x,3}$	Second and third order cumulants of input, respectively
$S_x(.)$, $S_y(.)$	Power spectra of system input and output, respectively
$B_x(.)$, $B_y(.)$	Bispectra of system input and output, respectively
$H(.)$	Frequency response function (FRF)
$h(t)$	Impulse response function
$y_n(t)$	The nth order Volterra output
$h_n(\tau_1,...,\tau_n)$	The nth order Volterra kernel
$E\{.\}$	Expected value operator
$H_n(\Omega_1,...,\Omega_n)$	The nth order generalized frequency response function
$\mathcal{F}[.]$	Fourier transform operator
$\delta(.)$	Dirac delta function
$\sigma(.)$	Integral field
$\otimes$	Kronecker product

By now, the bispectrum has become a useful and practical tool for non-Gaussian signal analysis and has been widely used in a good number of detection applications including biomedicine [4,5], fluid mechanics [6], plasma physics [7], geophysics [8], economics [9], engineering structures [10–13], mechanical systems [14–16], radar signals [17,18] and acoustic signals [19], etc. Besides great research efforts in exploiting the potential of bispectrum in practical applications, efforts have also been made to develop a variety of methods to estimate the bispectrum from the signals. Two comprehensive reviews about the bispectrum estimation methods were contributed by Nikias and Raghuveer [20,21].

Just as stated by Nichols and his colleagues in [22,23], despite increasing literature about the applications of bispectrum, there are very few available references [24] dedicating to the bispectrum for nonlinear system outputs, partly because deriving the bispectrum for nonlinear system outputs is much more difficult, compared to linear systems. By approximating nonlinear systems responses with convergent Volterra series, Nichols et al. [22,23,25] have successfully derived analytical expressions for the bispectra of the output responses of the nonlinear systems subjected to jointly Gaussian input and non-Gaussian input, respectively. In their studies, the second-order Volterra series was adopted, and so the results are usually valid for quadratic nonlinear systems. In addition, Nichols and his colleagues [26] have also determined analytical expression for trispectrum where cubic nonlinearity was taken into account. In this study, by modeling the nonlinear systems with Volterra series of arbitrary order, the method initiated by Nichols et al. [22] is extended to study the bispectrum for a wide class of polynomial nonlinear systems, not limited to the quadratic ones. A more general analytical expression is derived for the bispectrum calculation for polynomial nonlinear systems subjected to zero mean Gaussian excitations (no requirement to be spectrally white). Furthermore, based on the expression, an explicit relationship was established as polynomial function between the nonlinear characteristic parameters and the bispectrum.

2. Bispectrum analysis

2.1. Definition

For a stationary random signal $x(t)$, the second and third order cumulants [2] are defined as

$$C_{x,2}(\tau)=E\{x(t)x(t-\tau)\} \quad \text{and} \quad C_{x,3}(\tau_1,\tau_2)=E\{x(t)x(t-\tau_1)x(t-\tau_2)\}, \tag{1}$$

respectively, where τ_i is the time delay between successive observations. Taking Fourier transform on the cumulants above yields the power spectrum and the bispectrum, i.e.,

$$S_x(\Omega)=\int C_{x,2}(\tau)\exp(-j\Omega\tau)d\Omega \tag{2}$$

$$B_x(\Omega_1,\Omega_2)=\int\int C_{x,3}(\tau_1,\tau_1)\exp(-j(\Omega_1\tau_1+\Omega_2\tau_2))d\Omega_1 d\Omega_2 \tag{3}$$

Assume that $y(t)$ is the output of a linear system subjected to input $x(t)$, i.e.,

$$y(t)=\int h(\tau)x(t-\tau)d\tau \tag{4}$$

where $h(t)$ is the impulse response function of the system, then by the well-known linear system theory it follows that

$$S_y(\Omega)=S_x(\Omega)\left|H(\Omega)\right|^2 \tag{5}$$

$$B_y(\Omega_1,\Omega_2)=B_x(\Omega_1,\Omega_2)H(\Omega_1)H(\Omega_2)H^*(\Omega_1+\Omega_2) \tag{6}$$