Mechanical Systems and Signal Processing 35 (2013) 52–68

Contents lists available at SciVerse ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Modal parameter estimation by combining stochastic and deterministic frequency-domain approaches

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ARTICLE INFO

Article history:
Received 3 August 2011
Received in revised form
14 December 2011
Accepted 30 August 2012
Available online 27 September 2012

Keywords:
Modal parameter
Frequency-domain
Stochastic approach
Deterministic approach
Modal analysis

ABSTRACT

The poly-reference Least squares Complex Frequency-domain (pLSCF) estimator—commercially known as the LMS PolyMAX estimator—is used intensively in modal analysis applications nowadays. pLSCF is non-iterative (deterministic) and relatively accurate modal parameter estimation algorithm. This algorithm has several advantages: it is polyreference, fast, numerically stable for large-bandwidth with high-model order analysis, and yields very clear stabilization diagrams even with highly noisy FRFs measurements. One drawback of the pLSCF-estimator is that it yields a poor damping estimates especially for highly damped and weakly excited modes when the FRFs are very noisy.

In this contribution, an approach will be proposed to improve the accuracy of pLSCF estimator and in particular, the damping estimates in case of high noise level. The new proposed approach is a combined stochastic-deterministic frequency-domain algorithm. In this approach, a 2-step procedure is introduced to improve the damping estimates while maintaining the very clear stabilization diagrams. In the first step, a parametric Maximum Likelihood smoothing approach, which is the stochastic part, is used to remove the noise from the data and in the second step, the pLSCF estimator, which is the deterministic part, is applied to the smoothed data resulting in improved (damping) estimates.

The presented algorithm is able to maintain the benefits of pLSCF and at the same time leads to an improvement of the damping estimates in highly damped and very noisy cases. In addition, the new procedure properly deals with uncertainty on the measurements where the data variance due to measurement noise is taken into account.

The procedure is illustrated and tested by using simulated as well as experimental data. The presented procedure to process a highly damped noisy vibration data leads to very accurate estimates in comparison to the traditional pLSCF estimator.

1. Introduction

In the field of modal parameter estimation, the requirement for reliable extraction of modal parameters such as eigenfrequencies, damping ratios and mode shapes from a huge set of noisy FRFs is still a big issue. Over the last decades,

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http://dx.doi.org/10.1016/j.ymssp.2012.08.025

a number of algorithms have been developed to estimate modal parameters from measured frequency or impulse response function data. The algorithms have evolved from very simple single degree of freedom (SDOF) techniques to algorithms that analyze data from multiple-input excitation and multiple-output responses simultaneously in a multiple degree of freedom (MDOF) approach.

In the class of MDOF algorithms, a major grouping is usually done based on the domain in which the data are treated numerically resulting in time-domain and frequency domain methods. Within the domain of modal analysis it is commonly known to prefer time-domain estimators for systems with a low damping (damping ratio $\zeta < 1\%$), while frequency-domain estimators are better for highly damped systems (damping ratio $\zeta > 2.5\%$).

Several MDOF MIMO frequency-domain algorithms are developed, like e.g. the Least squares Frequency Domain (LSFD), Eigensystem Realization Algorithm (ERA) frequency-domain and Frequency-domain Direct Parameter Identification algorithm (FDPI). The LSFD directly estimates the modal parameters by solving a non-linear optimization using the modal model formulation. Good starting values for the poles, mode shape and participation factors are required to reduce the number of iterations. This method as such never became popular, except in the case that the poles and participation factors are already estimated in a first step by e.g. Least-Squares Complex Exponential (LSCE), Ibrahim Time Domain (ITD), the LSFD reduces to a linear least squares problem in the unknown modal parameters (mode shapes) [1]. The ERA frequency-domain version starts from a complex block matrix with the FRFs as a primary data and estimates the modal parameters by the use of SVD and eigenvalue decomposition (EVD) [2].

Based on a left matrix fraction description (LMFD) in the Laplace domain, FDPI estimates the modal parameters from the FRFs [3,4]. In FDPI algorithm, the poles, mode shapes and participation factors are derived from the system matrices of a state model. FDPI can sometimes not achieve stability for physical poles when the analysis bandwidth is too board. FDPI is therefore better suited for narrow band modal analysis [5]. A large number of frequency-domain estimators are based on a matrix fraction description model and these estimators are based on the minimization of an equation error between the measured and the modeled FRF matrix which in general is non-linear-in-parameters. The quadratic-like cost function is then defined as the sum of the Forbenius norm of the squared error function for each angular frequency line.

The most straightforward method to minimize that cost function is to use the non-linear Least Squares approach and this requires an iterative algorithm like e.g. a Gauss–Newton approach. To avoid this non-linear optimization process, Levi [6] proposed a way to linearize the cost function resulting in a linear least squares problem. Based on this idea, a large number of linear least squares estimators have been described in literature during the last decades. The so-called Rational Fraction (Orthogonal) Polynomial (RFOP) was introduced in [7] for SISO systems.This method was extended to a global method (GRFOP) for SIMO identification [8]. In [9,10], a polyreference modal parameter estimation technique based on a linear least squares approach was presented. Since this technique is working in Laplace-domain a different set of orthogonal polynomials were used for the numerator and the denominator of the Right-Matrix Fraction Description model to improve the ill-conditioned matrices for the high-order systems.

One of the most famous implementation of the frequency-domain Linear Least squares estimators optimized for the modal parameter estimation is called Least Squares Complex Frequency-domain (LSCF) estimator. This estimator has been developed for handling modal data sets that are typically characterized by a large number of response DOFs, high modal density and a high dynamic range [11]. It was optimized both for the memory requirements and for computation speed. This estimator uses a discrete-time common denominator transfer function parameterization results in well-conditioned Toeplitz structured matrices that can be constructed using the FFT algorithm. LSCF estimator allows a fast construction of the stabilization chart for an increasing model order. In [12], the LSCF estimator is extended for a polyreference case where a polyreference LSCF (pLSCF), commercially known as the PolyMAX estimator, is proposed. While the LSCF estimator uses a common denominator model, the P-LSCF estimator uses a Right Matrix Fraction Description (RMFD) model. The interest in using a RMFD model can be explained by the growing use of multiple input test setups. The main advantages of the pLSCF are its speed and the very clear stabilization chart it yields also in the case of noise-contaminated FRFs. However, the damping estimates associated with some stable poles decrease with increasing noise levels and this situation becomes worth for poorly excited modes and this is the big disadvantage of pLSCF estimator.

All the previously discussed estimators belong to the class of so-called deterministic algorithms, which in essence are curve-fitting techniques. By taking knowledge about the noise on the measured data into account in the cost function, it is possible to derive estimators with significant higher accuracy compared to the ones developed in the deterministic framework. This class of estimators is called stochastic estimators which most of them starts from the Fourier transform of the input (force) and response time sequences measured using a periodic excitation signal derived in an errors-in-variables framework, from which a sample mean and sample variance can be derived [13]. Based on this information, the model parameters can be derived using the so-called frequency-domain Maximum Likelihood Estimator (MLE) developed in [14] and extended to multivariable systems in [15]. In the same stochastic framework, Weighted Total Least squares algorithms with nearly maximum likelihood properties were developed for SISO [16] and MIMO [17] system identification. Although these stochastic algorithms feature nice statistical properties, the computational burden related to the iterative character and the initial formulation of the algorithms has been a barrier to apply these algorithms for commercial modal parameter estimation software. In [18], a multivariable implementation for frequency-domain Maximum Likelihood estimator, based on a common denominator transfer function model, has been given. This contribution aimed to optimize the Maximum Likelihood identification technique with respect to its drawbacks that are the memory requirements and the computation speed. With this technique, the modal parameters can be estimated accurately together with their confidence intervals in