Mechanical Systems and Signal Processing 35 (2013) 127–136

Contents lists available at SciVerse ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

A KIS solution for high fidelity interpolation and resampling of signals

J. Antoni [a,*], M. Eltabach [b]

[a] Laboratory Vibrations and Acoustics, University of Lyon, F-69621 Villeurbanne, Cédex, France
[b] CETIM, 52, Avenue Félix—Louat—BP 80067, 60304 Senlis, Cedex, France

ARTICLE INFO

Article history:
Received 6 September 2011
Received in revised form
5 August 2012
Accepted 9 August 2012
Available online 5 September 2012

Keywords:
Interpolation
Resampling
Oversampling
Perfect-reconstruction filter-bank

ABSTRACT

A keep-it-simple (KIS) solution is introduced that interpolates a signal up to an arbitrary accuracy. It consists of interpolating the complex envelopes at the output of a perfect-reconstruction filter-bank: if K subbands are used, the proposed interpolation scheme is shown to have a similar figure of merit as if the signal was initially oversampled by factor K at the acquisition stage, yet without the storage burden implied by the latter method.

1. Introduction

The interpolation and resampling of signals is a necessary step in many noise and vibration applications. This is for instance customary for order analysis and order tracking [1–4], time-synchronous averaging [5,6], cyclostationary analysis [7,8], torsional vibration and transmission error computation [9–12], and more generally in any application where the signal of interest (SOI) is resampled from one domain (typically time) to another domain (e.g., angle) [13,14]. Any resampling procedure is yet prone to introduce significant errors, which are important to control. Usual solutions to reduce interpolation errors (IE) are essentially of two types. The first one uses sophisticated interpolation schemes, like high-degree polynomials that closely approximate the ideal – but infinitely oscillating – cardinal sine. This requires specific algorithmic implementations which preclude the use of "off-the-shelf" and numerically optimised algorithms such as the popular cubic splines [15]. The second type of solution consists of oversampling the SOI (above twice its maximum frequency) during the initial acquisition stage and then to apply a standard (low-order) interpolation scheme [16]. Although this solution is usually favoured due to its simplicity, it unfortunately requires storing a large quantity of data that may quickly become prohibitive.

A simple solution is described herein which benefits from the advantages of the "over-sampling" method, yet without having to oversample the SOI at the acquisition stage.

* Corresponding author. Tel.: +33 4 72 43 71 83; fax: +33 4 72 43 72 37.
E-mail addresses: jerome.antoni@insa-lyon.fr (J. Antoni), mario.eltabach@cetim.fr (M. Eltabach).

0888-3270/$ - see front matter
http://dx.doi.org/10.1016/j.ymssp.2012.08.017

2. Interpolation, oversampling, and subband decomposition

2.1. Problem statement

This first subsection aims at stating the issue at hand and introducing the notations. Let $x(nT_s)$ be the SOI, initially sampled at time instants nT_s, $n=0,\ldots,N-1$, with Shannon's rate $F_s=1/T_s$; the object is to interpolate it at time $t\neq nT_s$. Most interpolation methods (e.g., splines, polynomials) can be formulated explicitly or implicitly as the convolution of the sampled signal with some (continuous-time) interpolating kernel $\varphi(t)$ [17], such that

$$\hat{x}(t)=T_s\sum_n x(nT_s)\varphi(t-nT_s) \tag{1}$$

is an estimate of $x(t)$. In general, the interpolating kernel $\varphi(t)$ is returned by interpolating the discrete delta, that is

$$\varphi(t)=T_s\sum\nolimits_n \delta_{n,0}\varphi(t-nT_s) \tag{2}$$

where $\delta_{n,0}=T_s^{-1}$ when $n=0$ and 0 otherwise. According to the celebrated Shannon sampling theorem, the ideal interpolating kernel that allows perfect reconstruction (PR) of a band-limited signal ($f_{\max}\leq F_s/2$) is the cardinal sine (sinc) function

$$\varphi_{\text{sinc}}(t)=\frac{\sin(\pi F_s t)}{\pi t}. \tag{3}$$

Unfortunately, the convolution with $\varphi_{\text{sinc}}(t)$ in Eq. (1) is unrealisable because it is of infinite duration. Indeed, any practical interpolation scheme boils down to approximating the latter in a finite number of steps. The IE of a given interpolation scheme is then completely controlled by the departure of the actual interpolating kernel from the ideal sinc function. In the frequency domain, this is reflected by the distance of $\Psi(f)=F\{x(t)\}$, the Fourier transform of $\varphi(t)$, to the ideal low-pass filter

$$\mathcal{F}\{\varphi_{\text{sinc}}(t)\}=\Pi_{F_s/2}(f)\equiv\begin{cases}1 & |f|\leq F_s/2\\ 0 & \text{otherwise}\end{cases}. \tag{4}$$

As illustrated in Fig. 1, a simple but very efficient solution to reduce the IE is to initially oversample the SOI (i.e., set a sampling rate F_s several times – say K – greater than $2f_{\max}$) in order to confine the signal spectrum inside the region where $\Psi(f)\approx\Pi_{f_{\max}}(f)=1$ and disregards regions where $|\Psi(f)-\Pi_{f_{\max}}(f)|$ is large. The price to pay, however, is a significant loss in storage capacity since a signal oversampled by factor K involves K times as many data as is strictly necessary. This may be redhibitory in many applications. The next subsection demonstrates how this limitation can be alleviated by means of a subband decomposition of the SOI.

2.2. Subband interpolation

From now on, let us comply with the constraint of a critically sampled SOI, $F_s=2f_{\max}$.

The key idea is to achieve the same performance as the direct oversampling method by interpolating K over-sampled subbands at the output of a PR (constant bandwidth) filter-bank. In order to introduce the idea, some standard but important results on PR filter-banks are first required.

2.2.1. Perfect reconstruction filter-banks

Specifically, let us partition the frequency axis $[-F_s/2;\ F_s/2]$ into $2K$ intervals with mirror symmetry at $f=0$, as illustrated in Fig. 2. This yields K subbands of a filter-bank indexed as $k=1,\ldots,K$, on the positive frequency interval $[0;\ F_s/2]$. The output, x_k, to subband k in the filter-bank is obtained as follows. Let us define $w(nT_s)$, the prototype impulse response of the filter-bank with bandwidth $F_s/2K$. Then,

$$x_k(nT_s)=(w(nT_s)e^{j2\pi f_k nT_s})\otimes x(nT_s) \tag{5}$$

(where $u(nT_s)\otimes v(nT_s)\equiv T_s\sum_k u((n-k)T_s)v(kT_s)$ stands for the convolution product between two discrete sequences u and v) denotes the band-pass version of the SOI through the kth subband with central frequency $f_k=(k-1/2)F_s/2K$. In a PR

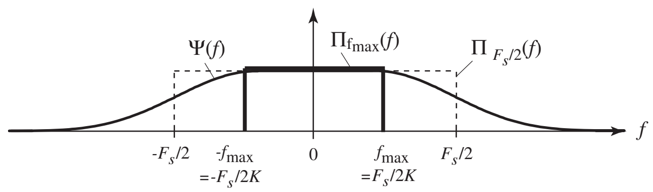

Fig. 1. Spectrum of the interpolation kernel, $\Psi(f)$, with initial oversampling by factor $K=F_s/2f_{\max}$.