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Estimation of nonparametric noise and FRF models for multivariable systems—Part I: Theory

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ABSTRACT

This series of two papers presents a method for estimating nonparametric noise and frequency response function models of multivariable linear dynamic systems excited by arbitrary inputs. It extends the results of Schoukens et al. (2006) [1] and Schoukens and Pintelon (2009) [2] from single input, single output systems with known input and noisy output observations (= output error problem), to multiple input, multiple output systems where both the input and output are disturbed by noise (= errors-in-variables problem). In Part I, the theory is developed for linear dynamic multivariable output error problems. The results are supported by simulations. A detailed comparison with the classical spectral analysis based on correlation techniques shows that the proposed procedures are more robust. In Part II (Pintelon et al., 2009) [3], the method first is applied to nonlinear systems, and parametric identification within a generalized output error framework. Next, it is extended to handle errors-in-variables problems, and identification in feedback. Finally, it is illustrated on four real measurement examples.

1. Introduction

Although most real life systems behave to some extend nonlinearly, linear dynamic models are very useful to understand, control, and predict physical processes, and to design (new) products [4–7]. A first step in the construction of a parametric transfer function model for the system is the measurement of the frequency response function (FRF) and its uncertainty (variance). In this paper we focus on the measurement of the frequency response function and the output noise variance in the frequency band of interest. Note that the knowledge of the noise variance is as important as the FRF value itself: the noise variance is used to generate uncertainty bounds on the FRF with a given confidence level [4–9], and is used as nonparametric weighting for the identification of the system transfer function model [7,10–14].

According to the nature of the excitation signal, one can distinguish between two noise measurement procedures. If the input is arbitrary (random), then it is assumed that the input is known exactly (= output error stochastic framework), and the output noise variance is obtained via spectral analysis (also called the H_1 method) or coherence techniques [4,5,8,15,16]. Other methods such as the correlogram or the $H_{\exp}$ estimator using an exponential window exist for suppressing the leakage in FRF measurements [16]. They, however, assume that the leakage of the full data record can be neglected; an assumption that is not made here. Moreover, no noise covariance estimates are available. If the input is periodic, then both input and output observations can be noisy (= errors-in-variables stochastic framework), and the input–output noise

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(co-)variances are obtained via the sample (co-)variances over consecutive input–output periods of the steady state response [7,11,12,14,17–19]. In this series of two papers we handle the arbitrary input case for multivariable systems for both the output error (Part I) and the errors-in-variables (Part II) case.

The major problem in estimating the nonparametric frequency response function and noise covariance matrix using arbitrary excitations is the suppression of the system and noise leakage errors. These errors are introduced when transforming a finite number N of time domain samples to the frequency domain via the discrete Fourier transform (DFT). Spectral analysis methods handle this problem via time domain windowing. To reduce the noise on the estimates, the record of N samples is divided into M subrecords of length N/M, which decreases the frequency resolution from f_s/N to Mf_s/N, and the results are averaged over the M subrecords. Hence, choosing M is making a trade-off between on the one hand the leakage elimination and the frequency resolution (the larger M, the larger the leakage errors and the smaller the frequency resolution), and on the other hand the noise suppression (the variance of the estimates decreases by M). Part I of the series of two papers presents a new method for nonparametric estimation of the FRF and the noise covariance matrix of multivariable systems. The basic assumption made is that the system and noise transfer functions are smooth functions of the frequency that can locally be approximated by a low degree polynomial. This so-called local polynomial approach has maximal frequency resolution f_s/N, and suppresses much better the system and noise leakage errors, while maintaining a useful noise averaging effect that is at least as good as that of the spectral analysis methods.

The major contributions of Part I of this series of two papers are:

1. The generalization of the local polynomial approach in [1,2] to multi-input, multi-output (MIMO) systems.
2. The extension of the bias analysis in [1,2] of the local polynomial frequency response function (FRF) and of the noise covariance estimates: the bias expressions are given as a function of the system and noise interpolation errors (= spectral errors resulting from the polynomial interpolation of, respectively, the system and noise dynamics over neighboring frequencies), and the system and noise leakage errors (= spectral errors resulting from the finite measurement time of, respectively, the input–output signals and the input–output noise).
3. The robustification of the local polynomial estimate of the noise covariance matrix to lack of excitation in the frequency band of interest.
4. The extension of the bias analysis in [1,2] (single-input, single-output), [4,8] (multiple-input, single output), and [5] (multiple-input, multiple-output) of the spectral analysis estimates of the FRF and noise covariance matrix using the diff [20], half-sine [22,23], and Hanning windows: the bias expressions are given as a function of the system and noise interpolation errors, and the system and noise leakage errors.

Part I is organized as follows. First, Section 2 defines the problem to be solved and states the assumptions made concerning the disturbing noise and the random excitation. Next, the local polynomial approach is extended to multivariable systems in Section 3. Further, Section 4 studies the properties of the spectral analysis methods with the rectangular, the diff, and the optimal half-sine windows; and shows that diff window performs equally well as the half-sine window. How to calculate uncertainty bounds on the FRF estimates is handled in Section 5. A theoretical comparison between the local polynomial approach and the spectral analysis methods is given in Section 6, while a numerical comparison of the performance can be found in Section 8. The problem of handling operational data is discussed in Section 7. Finally, some conclusions are drawn (Section 9).

2. Problem statement and assumptions

Consider the linear dynamic multivariable system of Fig. 1 with n_u inputs and n_y outputs. The arbitrary (random) excitation $u(t)$ is assumed to be known and the output $y(t)$ is disturbed by filtered (band-limited) white noise $v(t)$ (= output error framework). The input–output discrete Fourier transform (DFT) spectra $U(k)$, $Y(k)$ of N samples of the

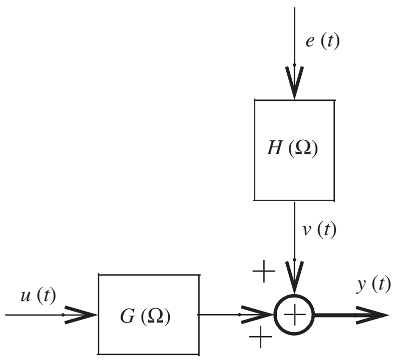

Fig. 1. Linear dynamic system with known input $u(t)$ and noisy output $y(t)$. The output noise $v(t)$ is written as filtered (band-limited) white noise.