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Adaptive regularization parameter optimization in output-error-based finite element model updating

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ABSTRACT

In finite element (FE) model updating, regularization methods are required to alter the ill-conditioned system of equations towards a well-conditioned one. The present study addresses the regularization parameter determination when implementing the Tikhonov regularization technique in output-error-based FE model updating. As the output-error-based FE model updating results in a nonlinear least-squares problem which requires iteration for solution, an adaptive strategy that allows varying value of the regularization parameter at different iteration steps is formulated, where the optimal regularization parameter at each iteration step is determined based on the computationally efficient minimum product criterion (MPC). The performance of MPC in output-error-based FE model updating is examined and compared with the commonly used L-curve method (LCM) and the generalized cross validation (GCV) through numerical studies of a truss bridge using noise-free and noise-corrupted modal data. It is shown that MPC is effective and robust in determining the regularization parameter compared with the other two methods, especially when noise-corrupted data are used. The adaptive strategy is more efficient than the fixed strategy that uses a constant value of the regularization parameter throughout the iteration process.

1. Introduction

Finite element (FE) model that can accurately represent the physical behavior of a structure is important in the disciplines of structural design and analysis, structural health monitoring, and structural control [1–3]. For example, in order to make a reliable prediction on the ultimate load-carrying capacity of a structure, an adequate FE model of the structure is desired. Despite the high sophistication of FE modeling, practical applications often reveal considerable discrepancies between analytical predictions and experimental results, which may originate from the uncertainties in simplified assumptions of geometry configuration, inappropriate values of material parameters, and inaccurate boundary conditions. Thus, the analytical model should be adjusted to coincide with the experimental results. In practice, the verification and updating of analytical models is mainly based on comparing experimentally obtained modal parameters with the analytical ones by means of FE model updating procedures.

A number of methods have been developed for correcting the FE models of structures so as to reproduce as closely as possible the experimental modal parameters [4–8], and they are commonly categorized as the input-error-based FE model updating approach and the output-error-based one. The former approach generally reduces model updating to solving

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a linear system of equations to obtain the updating parameters; whereas the latter approach gives rise to a nonlinear system of equations to solve for the updating parameters. Despite a lot of research efforts made, one of the critical issues that remain for model updating problems is how to deal with the resulting ill-conditioned system of equations [9]. It is known that model updating with the use of experimental modal parameters often leads to an ill-posed problem, where the existence and uniqueness of solution is not assured and numerical instability is likely to take place in the course of solution process [10,11]. The situation is further complicated by measurement noise as small measurement noise could be amplified by the ill-conditioning, leading to totally erroneous solution and divergence in iteration.

There have been some investigations attempting to deal with the ill-conditioning in FE model updating by using regularization methods in which the regularization parameter was determined by trial-and-error [12–16]. Anonymous selection of the regularization parameter is a critical issue for reliable implementation of the regularization methods. D'Ambrogio and Fregolent [17] applied truncated singular value decomposition (SVD) as a method of regularization to reduce the ill-conditioning in model updating, and the truncation parameter (a kind of regularization parameter) was chosen by simultaneous minimization of the natural frequency error and the response residual error. Ahmadian et al. [18] advocated the use of L-curve method (LCM) and generalized cross validation (GCV) for determining the regularization parameter in FE model updating. Ziaei-Rad and Imregun [19] studied a number of regularization methods and concluded that determination of the regularization parameter using LCM was straightforward. The above investigations regarding the regularization methods were made in connection with the input-error-based FE model updating approach, without needing iteration in solution. Mares et al. [20] explored a robust estimation method and the Tikhonov regularization method for the output-error-based FE model updating by using only the modal frequencies, and applied an uncertainty bound model and LCM to determine the regularization parameters. Most recently, Titurus and Friswell [21] investigated in some detail the Tikhonov regularization along with LCM in output-error-based FE model updating by using the experimental modal frequencies, and explored regularization in model updating from a geometric perspective. In most applications of model updating for model refinement in particular for structural damage identification, it is generally desirable to incorporate both the measured modal frequencies and mode shapes. It is presumable that the incorporation of mode shapes in model updating algorithms makes the system of equations more ill-conditioned as the magnitudes of modal frequencies and mode shapes often deviate in several orders [22].

This paper addresses the adaptive regularization parameter optimization when applying the Tikhonov regularization technique for output-error-based FE model updating. As the output-error-based FE model updating results in a nonlinear least-squares problem requiring iteration for solution, an adaptive strategy that allows varying value of the regularization parameter at different iteration steps is formulated. In recognizing that the adaptive strategy requires calculating the regularization parameter at each iteration step, a computationally efficient method on the basis of minimum product criterion (MPC) is explored to determine the optimal regularization parameter at each linearization step of the iteration process. Numerical studies of output-error-based FE model updating of a truss bridge using experimental modal frequencies and mode vectors are provided to compare the efficiency and accuracy of the adaptive strategy and the strategy adopting a fixed value of the regularization parameter throughout the iteration process. The performance of MPC in determining the optimal regularization parameter is also examined and compared with the commonly used LCM and GCV in both noise-free and noise-corrupted cases.

2. Output-error-based FE model updating

In system identification by means of FE model updating, the identification of structural parameters is formulated in an optimization problem where the structural parameters are sought so that the updated FE model can reproduce as closely as possible the experimentally obtained modal parameters. To this end, the objective function in the optimization problem, measuring the error between analytical and experimental modal parameters, is defined as follows

$$J_\theta = \boldsymbol{\varepsilon}^{\mathrm{T}}\mathbf{W}\boldsymbol{\varepsilon} = \left\|\mathbf{W}^{1/2}(\tilde{\mathbf{z}} - \mathbf{z}(\boldsymbol{\theta}))\right\|_2^2. \tag{1}$$

where $\boldsymbol{\varepsilon}$ is the output error of modal parameters; $\tilde{\mathbf{z}}$ and $\mathbf{z}(\boldsymbol{\theta}) \in R^n$ are the experimental and analytical modal parameter vectors with $n = n_f \times (n_m + 1)$; n_f and n_m are the number of experimental modal frequencies and the number of measuring points for each mode shape, respectively; $\boldsymbol{\theta} \in R^m$ is a vector of m updating parameters; $\mathbf{W}$ is a diagonal weighting matrix; and the subscript T denotes matrix/vector transpose. In order to obtain a unique solution, the number of known modal parameters n should be not less than the number of unknown updating parameters m. As the modal parameters $\mathbf{z}$ are generally nonlinear functions in terms of the unknown updating parameters $\boldsymbol{\theta}$, the output-error-based optimization problem defined by Eq. (1) is essentially a nonlinear least-squares problem.

It has been known that appropriate setting of the weighting matrix in Eq. (1) is important to improve the updating results [5]. Therefore, the relative weights for modal frequencies and mode shapes should be chosen carefully. As modal frequencies are obtained more accurately than mode shapes in practice, more weights should be placed on eigenvalues than on eigenvectors. In this study, the weights for eigenvalues are taken as unit and the weights for eigenvectors are taken as 0.1 [23].

The nonlinear least-squares problem in Eq. (1) can be iteratively solved using a gradient-based optimization method. When such a method is applied, the Jacobian matrix (or the sensitivity matrix) needs first to be calculated