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Relationship between state-space and ARMAV approaches to modal parameter identification

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Abstract

In this paper we study the relationship between state-space and autoregressive moving average vector (ARMAV) approaches for the problem of estimating the modal parameters of a vibrating system. Using only the singular value decomposition of the block Hankel matrix of covariances, it is shown that these two methods give identical modal parameters in the cases where this block Hankel matrix has full row rank.

Keywords: ARMAV model; State-space model; Companion matrix; State matrix

1. Introduction

Modal analysis has been used as a tool for analytical model validation, structure modification, response simulation in optimal design and to analyse the behaviour of a structure excited by a known or an unknown force. The use of experimental modal analysis has increased drastically with the development of the fast Fourier transform (FFT) algorithm and the advent of digital computers. However, the FFT algorithm has its own inherent problems such as those associated with leakage, bias and variance of spectral estimates. Time-domain methods have been developed to overcome these problems and to obtain the modal characteristics of vibrating systems. The modal parameters can be identified directly from measured data without performing any domain change and without applying any time window. Two time-domain approaches are analysed in this paper: the autoregressive moving average vector (ARMAV) approach and the state-space approach based on the stochastic realization algorithm. These two approaches have been applied to estimate the modal parameters of structures in real operational conditions [1–7]. In these cases, the exciting forces are unknown and thus the modal identification has to be carried out based on the responses only. The outputs, or responses, are the direct records of the sensors that are installed at several locations in the structure. With the ARMAV approach, the modal parameters are extracted by solving a standard eigenvalue problem of the companion matrix containing the autoregressive (AR) coefficients. With the state-space approach the modal parameters

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are obtained by solving a standard eigenvalue problem of the discrete state matrix also called the transition matrix.

In this paper, we describe briefly these two approaches and establish the relationship between the companion matrix and the transition matrix. It is shown that these two matrices give the same eigenvalues, and the same modal parameters, in the cases where the block Hankel matrix of covariances has full row rank. A generalized weighted concept of the ARMAV and state-space approaches is also analysed. An equivalent relation between ARMAV and state-space models is known since a long time, and the originality of the paper consists in the use of the singular value decomposition (SVD) of the block Hankel matrix of covariances to show the relationship between ARMAV and state-space approaches. Such a demonstration is conceptually and mathematically simple.

2. The ARMAV approach

The difference equation for an ARMAV time series is [1,6]

$$y_{k+f} - \sum_{j=1}^{f} \alpha_j y_{k+f-j} = \sum_{j=1}^{f} \beta_j \mathrm{e}_{k+f-j}. \tag{1}$$

The left side of this equation is the vector AR part and the right side is the vector moving average (MA) part. The AR part describes the system dynamics and contains all the modal information of the vibrating system, while the MA part is related to the external noise as well to the excitation. If m is the number of sensors, y_k is an $(m \times 1)$ vector of observations at time k: $y_k = [y_{1k}, y_{2k}, \ldots, y_{mk}]^{\mathrm{T}}$, where superscript T denotes the transpose. Also, e_k is an $(m \times 1)$ zero-mean vector white noise process, α_j's are the AR parameter matrices $(m \times m)$ and β_j's are the MA parameter matrices $(m \times m)$. The multi-dimensional ARMAV representation (1) can be expressed as

$$\begin{bmatrix} y_{k+1} \\ y_{k+2} \\ \cdot \\ \cdot \\ y_{k+f} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 & . & 0 \\ 0 & 0 & I & . & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & . & I \\ \alpha_f & \alpha_{f-1} & . & . & \alpha_1 \end{bmatrix} \begin{bmatrix} y_k \\ y_{k+1} \\ \cdot \\ \cdot \\ y_{k+f-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & . & 0 \\ \beta_f & \beta_{f-1} & . & . & \beta_1 \end{bmatrix} \begin{bmatrix} \mathrm{e}_k \\ \mathrm{e}_{k+1} \\ \cdot \\ \cdot \\ \mathrm{e}_{k+f-1} \end{bmatrix}. \tag{2}$$

Only the AR parameter matrices are necessary in order to identify the modal parameters of the vibrating system. These AR coefficients form the $(mf \times mf)$ companion matrix α defined as

$$\alpha = \begin{bmatrix} 0 & I & 0 & . & 0 \\ 0 & 0 & I & . & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & . & I \\ \alpha_f & \alpha_{f-1} & . & . & \alpha_1 \end{bmatrix} \tag{3}$$

and our objective is to find this companion matrix that can be eigenvalue–eigenvector decomposed. The eigenvalues are related to natural frequencies and damping ratios of the system and the eigenvectors are related to the mode shapes.

Define the $(mf \times 1)$ and $(mp \times 1)$ future and past data vectors as $y_k^+ = [y_k^{\mathrm{T}},\ y_{k+1}^{\mathrm{T}}, \ldots,\ y_{k+f-1}^{\mathrm{T}}]^{\mathrm{T}}$ and $y_k^- = [y_k^{\mathrm{T}},\ y_{k-1}^{\mathrm{T}}, \ldots,\ y_{k-p+1}^{\mathrm{T}}]^{\mathrm{T}}$. Define also the $(mf \times 1)$ vector e_k^+ as $\mathrm{e}_k^+ = [\mathrm{e}_k^{\mathrm{T}},\ \mathrm{e}_{k+1}^{\mathrm{T}}, \ldots, \mathrm{e}_{k+f-1}^{\mathrm{T}}]^{\mathrm{T}}$ and the $(mf \times mf)$