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Haar wavelet for machine fault diagnosis

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Abstract

Continuous wavelet transform (CWT) is a kind of time–frequency analysis method commonly used in machine fault diagnosis. Unlike Fourier transform, the wavelet in CWT can be selected flexibly. In engineering application, there is a problem of how to select a suitable wavelet. At present, the selecting method mainly depends on the waveform similarity between the signal required to extract and the wavelet. This method is imperfect. For example, Haar wavelet possesses the rectangular waveform in its supporting field and dissimilarity to any component in the machine signal. It is rarely used in machine diagnosis. However, the time–frequency periodicity of Haar wavelet continuous wavelet transform (HCWT) should be useful in revealing the features in signals. In addition, Haar wavelets under different scales have good low-pass filter characteristic in frequency domain, particularly under larger scales, and that can allow HCWT to detect the lower frequency signal. These merits are presented in this paper and applied to diagnose three types of machine faults. Furthermore, in order to verify the effect of Haar wavelet, the diagnosis information obtained by HCWT is compared with that by Morlet wavelet continuous wavelet transform (MCWT), which is popular in machine diagnosis. The results demonstrate that Haar wavelet is also a feasible wavelet in machine fault diagnosis and HCWT can provide abundant graphic features for diagnosis than MCWT.

Keywords: Machine fault diagnosis; Continuous wavelet transform (CWT); Haar wavelet; Time-scale periodicity; Frequency characteristic

1. Introduction

Wavelet transform has an excellent time–frequency localised property. It can make the interested component submerged in an original signal become distinct under certain scales. So, it has become popular in machine fault diagnosis [1–4]. Unlike Fourier transform that only has a sin basis, continuous wavelet transform (CWT) can be performed based on many types of wavelet basis, which may lead to differences in results when using them to process the same signal. Therefore, an important problem when using wavelet transform is how to select a suitable wavelet. Selecting a wavelet with the highest feature detecting capability by comparing the detecting effects of different wavelets using the simulating signal is not available in machine diagnosis, because the simulation of the machine signal is difficult or even impossible under most machine running conditions.

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However, the conventional method of according to the waveform similarity between wavelet and interested signal is also imperfect for its neglect of wavelet frequency domain property. In order to prove the argument, the paper selects Haar wavelet as the analysis wavelet, because Haar wavelet possesses the rectangular waveform in its supporting field and dissimilarity to any component in the machine signal. Firstly, the frequency characteristics of some wavelets are discussed. Different wavelets have different filter characteristics in frequency domain. But Haar wavelet has good low-pass filter characteristic, particularly under larger scales. Then the time-scale periodicity of Haar wavelet continuous wavelet transform (HCWT) is explained mathematically and graphically. As the applications, Haar wavelet is used to diagnose the faults of three types of machines, which are rotor, gearbox and rolling bearing. Furthermore, Morlet wavelet, which prevailed in machine fault diagnosis, is selected to be a comparative wavelet. Both HCWT and Morlet wavelet continuous wavelet transform (MCWT) are applied to extract the fault information of the same machine signals.The results demonstrate that HCWT can extract the harmonic and the impulse features, which are the dominating components in machine fault signals and provide abundant graphic features for diagnosis than MCWT.

2. Common wavelet frequency domain characteristics

2.1. CWT

If there is a wavelet $\psi(t) \in L^2(C)$, where $L^2(C)$ is a complex function space where the functions are square integrable. For a time-variable signal $x(t)$, CWT's definition is

$$W_x(a,b) = \frac{1}{\sqrt{a}} \int x(t)\psi^*\left(\frac{t-b}{a}\right) \mathrm{d}t = \langle x(t), \psi_{ab}(t) \rangle, \tag{1}$$

where

$$\psi_{ab}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \tag{2}$$

are the shifted and dilated forms of the wavelet basis and similar to a set of variable window functions. a ($a>0$) is the scale parameter, b is the time parameter, $\psi^*(t-b/a)$ is the complex conjugate of $\psi(t-b/a)$. $\langle \bullet \rangle$ represents the inner product operator. The wavelet basis in CWT is only required to satisfy the following admissibility condition:

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{|\omega|} \mathrm{d}\omega < \infty, \tag{3}$$

where $\Psi(\omega) = \int \psi(t) e^{-j\omega t} \mathrm{d}t$. Expression (1) in frequency domain is

$$W_x(a,b) = \frac{\sqrt{a}}{2\pi} \int X(\omega)\Psi^*(a\omega) e^{j\omega b} \mathrm{d}\omega, \tag{4}$$

where $X(\omega) = \int x(t) e^{-j\omega t} \mathrm{d}t$. It can be seen that if $\Psi(\omega)$ is a band-pass filtering function with a concentrated amplitude–frequency characteristic, CWT will have the capability of frequency domain localised. From the view of frequency domain, a set of wavelets with different scales is equal to a set of filters. So, CWT provides the adaptive resolution for the time–frequency decomposition of a signal.

2.2. Wavelet frequency domain filter characteristic

Since the wavelets under different scales are equal to a set of filters in frequency domain, it is important to study how the filter characteristics affect the signal processing results. Fig. 1 shows the filter characteristics of several wavelets commonly used in machine diagnosis. These figures indicate that not all wavelets have the band-pass filter characteristic (see Table 1), and the changing trend and the separability of characteristic curves are also different. Naturally, the processing results with different wavelets are not the same. In addition, for Meyer, Mexican Hat and Morlet wavelets, the amplitudes of the characteristic curves degrade sharply