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A nonlinear monostable filter for bipolar pulse signal detection

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Abstract

We investigate a nonlinear bistable system as a nonlinear filter for bipolar pulse signal detection. This nonlinear system usually shows the phenomenon of stochastic resonance (SR) by adding noise. To thoroughly discuss the detection performance of the nonlinear system, we tune the system parameters to obtain the optimal detection performance. In this case, the system in fact is a monostable system, so here it is named as a nonlinear monostable filter. For quantifying observations of the nonlinear system output, we define the probability of detection error, and give the theoretical and numerical results. To investigate the detection performance of the monostable filter, it is compared with SR and the matched filter, respectively. We find that (a) the detection performance of the nonlinear monostable filter is better than that of the system exhibiting SR, (b) the detection performance of the matched filter surpasses that of the monostable filter in perfect condition of synchronisation, and (c) as the desynchronisation is introduced, with increasing desynchronisation the monostable filter catches up and even is superior to the matched filter.

Keywords: Nonlinear monostable filter; Signal detection; Stochastic resonance; Matched filter

1. Introduction

Nonlinear dynamical systems present rich potentialities for signal and information processing. One of the interesting phenomena which attract great attention of specialists on nonlinear dynamics is stochastic resonance (SR) (see [1,2] for overviews). A noisy nonlinear system exhibits SR if its performance is optimised by nonzero value of the noise intensity. One of the earliest and most studied systems for SR, is the nonlinear bistable system governed by a double-well potential. Since it was originally put forward in 1981, many researchers have further analysed the potentialities of such nonlinear systems for various fields [1,2]. Particularly, in signal processing some promising results were obtained [3–10], which are important for engineers, especially in situations where the possibilities of linear systems have been exhausted.

In most SR studies, the parameters of the bistable dynamical system are fixed and the noise is added to obtain a good performance by producing the cooperative effect between a stochastic-subjected dynamical system and a signal. In this paper, for the bistable system used as a nonlinear detector, instead of tuning the level of the noise with a fixed system, we tune the system parameters in order to optimise the detection

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performance at a fixed noise level, which is somehow the classical way of optimising a tunable processing device. We shall complement the analysis of the nonlinear system for detection in the presence of a Gaussian band-limited white noise. Here, the nonlinear system is used for a detection task, in the sense of classical detection theory [11], and the assessment is made through the standard probability of detection error. To evaluate this probability for the nonlinear system, we calculate the dynamical solution of the Fokker–Planck equation (FPE) by an approximate method, and the theoretical results are checked by simulative experiments. By tuning the system parameters to the optimal detection performance, we find that the nonlinear bistable becomes a monostable system and the monostable system has a better detection performance than the bistable system, so in this paper the nonlinear filter is named as a monostable filter. We compare the monostable filter with SR and the matched filter, respectively. Such comparisons, which have never been explicitly undertaken, are important as a reference for a better appreciation of the potentialities of the monostable system.

2. The model

Consider the earliest system that SR was revealed, a nonlinear bistable dynamic system governed by the quartic potential. With the input signal and noise, it can be described as

$$\dot{x} = ax - bx^3 + s(t) + \Gamma(t), \quad a, b > 0, \tag{1}$$

where a and b are usually given in terms of system parameters, $s(t)$ is a pseudorandom bipolar pulse signal, and $\Gamma(t)$ denotes a zero mean Gaussian band-limited white noise with a noise intensity D. For a bipolar pulse signal, we introduce two hypotheses

$$\mathrm{H}_0: \ s(t) = -A \quad \text{with prior probability } P_0,$$

$$\mathrm{H}_1: \ s(t) = A \quad \text{with prior probability } P_1, \tag{2}$$

where $s(t)$, for $(n-1)T_b \leqslant t \leqslant nT_b$ and $n = 1, 2, \ldots, N$, has the pulse duration time T_b and pulse amplitudes $\pm A$. At each time multiple of the pulse duration T_b, the output of the system $x(t)$ is read by making a decision

$$\mathrm{D}_0: \ s(t) = -A \quad \text{if } x(t) < x_0,$$

$$\mathrm{D}_1: \ s(t) = A \quad \text{if } x(t) > x_0, \tag{3}$$

where x_0 is a decision level. In the above detection process, both tuning D (SR) and optimising parameters a and b can improve the detection performance of the nonlinear system. The performance will be measured by the probability of detection error

$$P_{er} = P_0 P(\mathrm{D}_1|\mathrm{H}_0) + P_1 P(\mathrm{D}_0|\mathrm{H}_1). \tag{4}$$

3. Quantitative observations of the nonlinear system output

Assume the signal levels are statistically independent and equiprobable (i.e., $P_0 = P_1 = \frac{1}{2}$), so Eq. (4) becomes

$$P_{er} = P_0 P(\mathrm{D}_1|\mathrm{H}_0) + P_1 P(\mathrm{D}_0|\mathrm{H}_1) = \tfrac{1}{2}[P(\mathrm{D}_1|\mathrm{H}_0) + P(\mathrm{D}_0|\mathrm{H}_1)]. \tag{5}$$

In each pulse duration T_b, system (1) is subjected to the constant amplitude $+A$ or $-A$, with an additive input Gaussian white noise (the band-limited noise can be approximate to white noise as the noise bandwidth is much larger than the bit rate $1/T_b$). Under these conditions, the statistically equivalent description for the corresponding probability density $\rho(x, t)$ is governed by the FPE

$$\frac{\partial \rho(x, t| \pm A)}{\partial t} = \frac{\partial}{\partial x}[V'_{\pm A}(x)\rho(x, t| \pm A)] + \frac{D}{2}\frac{\partial^2 \rho(x, t| \pm A)}{\partial x^2}, \tag{6}$$

where $V'_{\pm A}(x)$ is the derivation of the potential $V_{\pm A}(x)$ with respect to x, and

$$V_{\pm A}(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4 \mp A. \tag{7}$$