Advances in Engineering Software 47 (2012) 178–187

Contents lists available at SciVerse ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

Predicting torsional strength of RC beams by using Evolutionary Polynomial Regression

Alessandra Fiore [b,*], Luigi Berardi [b], Giuseppe Carlo Marano [a]

[a] Department of Environmental Engineering and Sustainable Development, Technical University of Bari, viale del Turismo 10, 74100 Taranto, Italy
[b] Department of Civil and Environmental Engineering, Technical University of Bari, via Orabona 4, 70125 Bari, Italy

ARTICLE INFO

Article history:
Received 19 April 2011
Received in revised form 19 July 2011
Accepted 2 November 2011
Available online 10 December 2011

Keywords:
Reinforced concrete beam
Evolutionary Polynomial Regression
Torsional strength
Building code
Theoretical model
Soft computing

ABSTRACT

A new view for the analytical formulation of torsional ultimate strength for reinforced concrete (RC) beams by experimental data is explored by using a new hybrid regression method termed Evolutionary Polynomial Regression (EPR). In the case of torsion in RC elements, the poor assumptions in physical models often result into poor agreement with experimental results. Nonetheless, existing models have simple and compact mathematical expressions since they are used by practitioners as building codes provisions.

EPR combines the best features of conventional numerical regression techniques with the effectiveness of genetic programming for constructing symbolic expressions of regression models. The EPR modeling paradigm allows to figure out existing patterns in recorded data in terms of compact mathematical expressions, according to the available physical knowledge on the phenomenon (if any). The procedure output is represented by different formulae to predict torsional strength of RC beam. The multi-objective search paradigm used by EPR allows developing a set of formulae showing different complexity of mathematical expressions as resulting into different agreement with experimental data.

The efficiency of such approach is tested using experimental data of 64 rectangular RC beams reported in technical literature. The input parameters affecting the torsional strength were selected as cross-sectional area of beams, cross-sectional area of one-leg of closed stirrup, spacing of stirrups, area of longitudinal reinforcement, yield strength of stirrup and longitudinal reinforcement, concrete compressive strength.

Those results are finally compared with previous studies and existing building codes for a complete comparison considering formulation complexity and experimental data fitting.

1. Introduction

In last decades a number of advancements has been observed in the field of reinforced concrete elements analysis; among them a number of remarkable research studies were aimed at overcoming problems in predicting final strength under different actions by using simple and effective formulations. Moreover differently from other situations, the torsional design of structural concrete elements is a complex problem which still remains an open question. In particular a more rational design formulation for RC structures under these specific actions still needs to be defined. The aim is to develop direct formulations which can be used without incurring the computational burden of numerical approaches like FEM. Such formulas, eventually, could be used for upgrading both existing best practices and current building codes. Moreover, such explicit formulations should be easily included into existing design codes, without significantly increasing the computational burden while improving prediction accuracy.

In this field (i.e. final torsional strength evaluation) there is an evident lack of efficient models to describe real physical behavior. In fact, for a given beam section, different current design codes might return quite different predictions which can vary by factors of more than two.

This is deeply different from other situations, such as the flexural strengths, whose predictions by the same codes may differ from each other by less than 10%. Actually, for flexure ultimate states some detailed models are used which take into account equilibrium, capability and non-linear stress–strain relationship of material. On the contrary, most of design equations for torsional strength derive directly from the equilibrium condition of the simple truss model theory proposed by Ritter and Morsch at the turn of the 20th century. This strong discrepancy is due to the difficulty in modeling the complex phenomena underlying the mechanical behavior of torsion by standard synthetic models.

* Corresponding author. Address: Politecnico di Bari, via Orabona 4, 70125 Bari, Italy. Tel.: +39 0805963525; fax: +39 0805963719.
E-mail address: a.fiore@poliba.it (A. Fiore).

0965-9978/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.advengsoft.2011.11.001

Nevertheless, several analytical and experimental studies have been reported in literature about torsional behavior of reinforced concrete (RC) members subjected to pure torsion or combination of torsion with other loadings, such as axial, shear and bending [1]. At present it is known that cross-sectional area of beam and its geometry, dimensions of closed stirrup, spacing of stirrups, cross-sectional area of one-leg of closed stirrup, yield strength of stirrup and longitudinal reinforcement and concrete compressive strength are the most important parameters involved. The effect of these variables on the torsional strength of RC beams has been extensively studied and several empirical approaches related to these variables have been investigated [2]. Test data are often used for validation, calibration or even development of above mentioned models.

However, although many experimental studies have been carried out to clarify the torsional behavior of RC beams, the estimation of the torsional strength of these structures still represents a complex task due to the high number of involved parameters [1].

In such a context, it is worthy to use some recently developed data analysis techniques aimed at discovering existing patterns in recorded data. A number of techniques has been proposed in recent years for identifying mathematical models of complex systems based on observed data [3]. Such techniques can be roughly ranked from white-box to black-box techniques depending on the level of prior information required/available. A white-box model is a system where all necessary information is available, i.e. the model is based on first principles (e.g. physical laws), variables are known and parameters have physical meaning. Conversely, black-box models do not require any prior information on the system in hand; they are data-driven or purely regressive models, for which neither functional form of relationships between variables nor numerical parameters are known and need to be estimated from training data. In between, there is a wide "palette" of gray-box models; they are conceptual models whose mathematical structure can be derived through conceptualization of physical phenomena or through simplification of known differential equations describing the phenomenon. These models usually need parameter estimation by means of input/output data analysis, although some information about underlying relationship is known.

White-box models, such as those developed for torsional ultimate strength prediction, have the advantage of describing the process being modeled using known mathematical relationships. However, the construction of white-box models can be difficult because the underlying mechanisms may not always be wholly understood, or because experimental results obtained in the laboratory environment do not correspond well to the prototype environment. Owing to these problems, approaches based on data-driven techniques are gaining considerable interest.

Among the existing data-driven techniques, Artificial Neural Networks (ANNs) and Genetic Programming (GP) are probably the most known. The first applications of Artificial Neural Network (ANN) date back to the early 1980s. Recently ANNs has been used as an effective tool to investigate different aspects of structural engineering, from analysis and design including constitutive laws of materials [4–6], to dynamics and structural damage [7,8]. A recent work [2] analyzed different ANNs for predicting the torsional strength of RC beams. The input parameters affecting the torsional strength were selected to be cross-sectional area of beams, dimensions of closed stirrups, spacing of stirrups, cross-sectional area of one-leg of closed stirrup, yield strength of stirrup and longitudinal reinforcement, steel ratio of stirrups, steel ratio of longitudinal reinforcement and concrete compressive strength. Each parameter was arranged in an input vector and a corresponding output vector included the torsional strength of RC beam.

However, in spite of their versatility as regression techniques, ANNs require the structure of a neural network to be identified *a priori* (e.g. model inputs, transfer functions, number of hidden layers, etc.). Furthermore, parameter estimation and over-fitting problems represent the principal disadvantages of model construction by ANN, as reported in [9]. These, in turn might result into lack of generalization capabilities of final models. Another difficulty with the use of ANNs is that they do not result into easily interpretable relationships which might help improve understanding of the physical phenomenon.

Genetic Programming (GP) is a modeling method that resorts to a population-based artificial intelligence technique to generate a structured representation of the system model. The model construction procedure mimics the natural evolutionary selection where the 'fittest' individuals (i.e. mathematical expressions of model) improve through successive generations. This technique allows a global exploration of the space of model expressions which can follow some user defined criteria.

On the one hand this strategy results into robust model search; on the other hand it potentially allows the user to gather additional information on system behavior by unveiling relationships between input and output data. Such features made the GP more appealing than ANNs for those context where the understanding of the system is still not completely known.

Among the GP methods, the so-called *symbolic regression* proposed by [10] is probably the most used. This technique uses the evolutionary search paradigm for developing explicit mathematical expressions of the model to fit a set of data points. However, the original GP method used to performing symbolic regression had some limitations since it does not perform a proper numerical estimation of model parameters (constants/coefficients) and it tends to produce expressions that grow in length during the evolutionary search [11]. Some notable attempts to mitigate those disadvantages have been reported for example by [12].

The Evolutionary Polynomial Regression (EPR) technique [13] has been introduced and continuously developed in the last years as an hybrid data-driven technique. EPR combines the effectiveness of evolutionary search for developing "transparent" and structured mathematical expression of input–output relationships with the advantage of classical numerical regression for parameter estimate. The EPR technique has been successfully applied to modeling a wide range of complex engineering problems including constitutive modeling of soils; stability of slopes; settlement of foundations; liquefaction of soils due to earthquake [14] and a number of other applications in Civil engineering [15–17]. A recent version of EPR, named EPR-MOGA, exploits Multi-Objective Genetic Algorithms (MOGAs) to search those model expressions which maximize accuracy of data and parsimony of mathematical expressions simultaneously. The main advantage of such approach is that EPR returns a set of explicit expressions with different accuracy to experimental data and different degree of complexity of mathematical structure of models. The analysis of such trade-off solutions between accuracy and complexity allows selecting those models which are better suited for specific applications.

This paper leverages such key features of EPR for developing explicit compact expressions to assess the ultimate torsional strength for RC beams.

To achieve these objectives, experimental data of 64 beams subjected to torsion are used. The just mentioned experimental data are obtained from [2], where the results of several tests performed by different authors such as Rasmussen and Baker [18], Koutchoukali and Belarbi [19], Fang and Shiau [20], and Hsu [21] are collected.

The experimental values of torsional strength are finally compared with both the relevant predictions of the main building codes, such as ACI-318-2005 [22], Eurocode-2 [23], TBC-500-2000 [24], CSA [25], BS8110 [26] and AS3600 [27], and the results obtained by EPR, in order to assess the efficacy of the proposed formulas.