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QML-RANSAC: PPS and FM signals estimation in heavy noise environments

Igor Djurović [a,b,*]

[a] University of Montenegro, Electrical Engineering Department, Cetinjski put bb, 81 000 Podgorica, Montenegro
[b] Institute for Cutting Edge Information and Communication Technologies, Džordža Vašingtona 66/354, Podgorica, Montenegro

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ABSTRACT

The QML-RANSAC estimator is proposed. It combines the quasi-maximum likelihood (QML) estimator with the random sample consensus (RANSAC). This technique can with reasonable calculation complexity work for lower the signal-to-noise ratio (SNR) than existing parametric estimators of polynomial phase signals (PPS) and nonparametric estimators of FM signals, i.e., it achieves lower SNR threshold than the state-of-the-art techniques in the field. Obtained results are better for about 3 dB with respect to the QML in term of the SNR threshold without increasing the mean squared error (MSE) above the threshold. The proposed estimator is tested on the PPS as a parametric estimator and for general FM signal estimation as a nonparametric estimator. An extension of the algorithm is proposed for multicomponent signals, as well.

1. Introduction

An important research topic in area of the nonstationary frequency modulated (FM) signal analysis and estimation is processing in the high noise environment [1–3]. It can be considered in a framework of the parametric estimation with assumed model of the signal phase or nonparametric estimation when model is not known in advance. Among parametric models the polynomial phase signal (PPS) is the most popular, while the instantaneous frequency (IF) (or less commonly phase) is estimated in the case of nonparametric estimators [4–6].

There is important recent progress in handling high noise environment in these fields. In the area of nonparametric estimators the Viterbi algorithm is applied to the time-frequency (TF) representations [2,8]. Other developments in this area include the improved intersection-of-the-confidence-interval algorithm for IF estimation [1], application to newborn seizures characterization [3], high-resolution TF representations for feature extraction of signal patterns [7], etc. Even more dynamic development can be observed in the area of parametric estimation with numerous tools developed over last 20 years including high-order ambiguity function (HAF), product HAF (PHAF), integrated generalized ambiguity function (IGAF), cubic phase function (CPF), hybrid (combination) of the CPF and HAF (CPF-HAF), least squares phase unwrapping (LSPU), and quasi maximum likelihood function (QML) [9–32]. Here, we want to emphasize the QML (quasi maximum likelihood) as a method that is giving excellent results for high-order PPS without significant loss of accuracy that is common for other parametric estimators. Also, it is important to stress that the QML can be applied to signals deviating from the assumed polynomial order [9]. The QML gives results that are close or on the Cramer Rao lower bound (CRLB) with low SNR threshold for a high-order PPS [9,10].

The crucial open question is not reducing the mean square error (MSE) but lowering the SNR threshold under reasonable calculation complexity (at least several orders of magnitude bellow direct search required for the ML estimators for higher-order PPS). Namely, the QML and the O'Shea refinement algorithm [22] offer methodology for reducing the MSE toward the CRLB above the SNR threshold. Alternatively, the Nelder-Mead approach can be used for the refinement [33]. Therefore, the main focus should be at lowering the SNR threshold with moderate (or at least reasonable) increase of complexity.

In the QML, rough estimate of PPS parameters is determined by interpolation of IF estimates obtained by using position of the short-time Fourier transform (STFT) maxima [9]. In a high noise environment some of the IF estimates are outliers causing inaccurate rough estimate. However, positions of outliers are unknown so it is important to consider some alternative sampling tools to avoid outliers in estimates.

In our research we are achieving such goal with the random

* Correspondence address: University of Montenegro, Electrical Engineering Department, Cetinjski Put bb, 81 000 Podgorica, Montenegro.
E-mail address: igordj@ac.me

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sample consensus (RANSAC) [34,35]. The RANSAC is used in estimation of features and parameters in various signal processing related disciplines [36,37]. The main advantage of the RANSAC algorithm in our application comes from the random selection of instants. In such a way, performing multiple random selections of the IF estimate samples it is possible to find combination of samples without outliers giving accurate rough estimate of signal parameters. In the second stage, the O'Shea refinement is applied to improve accuracy. It is also generalized for a case when precise parametric model is not known. In addition, we have demonstrated an algorithm modification for multicomponent signals.

Obtained results are excellent surpassing in the term of the SNR threshold all existing techniques including the Viterbi algorithm for nonparametric estimation and the QML for parametric estimation. Calculation complexity of this technique is increased with respect to the QML but it is paid-off by decreasing the SNR threshold for about 3–4 dB.

The paper is organized as follows. In Section 2, we are giving information about signal model employed and related estimators. In Section 3, the QML-RANSAC variant of the IF (phase) and PPS parameter estimator is proposed. Section 4 gives numerical examples demonstrating accuracy of the proposed technique. Section 5 concludes the paper.

2. Signal model and related techniques

2.1. Signal model

A monocomponent FM signal embedded in a white Gaussian noise:

$$x(n) = f(n) + \nu(n) = A\ \exp(j\phi(n)) + \nu(n), \tag{1}$$

is considered, where $\phi(n)$ is a signal phase while A is a signal amplitude. The PPS model follows when $\phi(n)$ is modeled as a polynomial function

$$\phi(n) = a_0 + \sum_{k=1}^{K} a_k n^k/k. \tag{2}$$

Commonly, in the case of nonparametric estimators it is assumed only that the phase derivatives are limited $|\phi^{(k)}(n)| < M_k$. Here, we consider that a noise environment is Gaussian white with variance σ^2. The goal is to estimate coefficients $\{a_k | k \in [0, K]\}$ for parametric case, and the IF (the first derivative of the signal phase $\omega(n) = \phi'(n)$ as the most important feature) for nonparametric estimators.

Multicomponent signal model addressed in Section 3.3 and numerical examples is given as

$$x(n) = \sum_{q=1}^{Q} A_q \exp(j\phi_q(n)) + \nu(n), \tag{3}$$

where phase $\phi_q(n)$ and phase coefficients are related as

$$\phi_q(n) = a_{0,q} + \sum_{k=1}^{K} a_{k,q} n^k/k. \tag{4}$$

2.2. QML algorithm

Search for all parameters of the PPS can be performed using the maximum likelihood (ML) approach

$$J(b_1, b_2, \ldots, b_K) = \left|\sum_n x(n)\exp\left(-j\sum_{k=1}^{K} b_k n^k/k\right)\right| \tag{5}$$

$$(\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_K) = \arg\max_{(b_1, b_2, \ldots, b_K)} J(b_1, b_2, \ldots, b_K). \tag{6}$$

However, search over K-dimensional parameter space is demanding for high K, $O(N^K)$, and it is unfeasible and some alternative techniques are required.

An approach for handling this issue is proposed recently and it is called the QML. Search in the QML is performed for a single parameter, i.e., window width in the STFT.

General description of this technique can be written as:

$$(\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_K) = (b_1(\hat{h}), b_2(\hat{h}), \ldots, b_K(\hat{h})) \tag{7}$$

$$\hat{h} = \arg\max_h J(h), \tag{8}$$

$$J(h) = \left|\sum_n x(n)\exp\left(-j\sum_{k=1}^{K} b_k(h) n^k/k\right)\right|. \tag{9}$$

where h represents the window width in the STFT over which the search is performed.

For each window width $h \in H$ the STFT is calculated

$$\mathrm{STFT}_h(n, \omega) = \sum_k x(n+k) w_h(k) \exp(-j\omega k),$$
$$n \in [-N/2 + h/2, N/2 - h/2), \tag{10}$$

where window function is defined as

$$w_h(k) = 1, \quad k \in [-h/2, h/2)$$
$$= 0, \quad \text{elsewhere}. \tag{11}$$

The IF is estimated using position of the STFT maxima:

$$\hat{\omega}_h(n) = \arg\max_\omega |STFT_h(n, \omega)|, \tag{12}$$

Rough estimation of the phase parameters is performed using polynomial interpolation of the IF estimates $\hat{\omega}_h(n)$. Denote these estimates as $\{c_k(h) | k \in [1, K]\}$. For details on the employed polynomial regression refer to [9] and see Section 3. Then, the considered signal is dechirped using rough estimates as $\hat{x}(n) = x(n)\exp\left(-j\sum_{k=1}^{K} c_k(h) n^k\right)$. This signal is low-pass FM of the same model as initial one with the same noise variance as in the input signal. In order to attenuate noise influence low-pass filtering can now be employed not affecting parameters of signal $\tilde{x}(n) = \mathrm{MAF}\{x(n)\}$, where MAF{} is the moving average filter (MAF) operator. Polynomial regression is then applied to the unwrapped phase of the filtered signal $\tilde{\phi}(n) = \mathrm{unwrap}(\mathrm{phase}(\tilde{x}(n)))$ and estimated coefficients denoted as $\{\Delta c_k(h) | k \in [0, K]\}$ are again obtained using polynomial interpolation. Estimate $b_k(h)$ follows as sum of results from the rough and fine stages:

$$b_k(h) = c_k(h) + \Delta c_k(h), \quad k \in [1, K] \tag{13}$$

$$b_0(h) = \Delta c_0(h). \tag{14}$$

As it is demonstrated in [9,10], this technique is able to achieve the MSE on the CRLB above the SNR threshold and the SNR threshold is low not exceeding 2 dB even for a high-order PPS (K=10 and more). However, in this research we want to investigate if it is possible to reduce the SNR threshold further. Note, that a combination of the STFT and phase unwrapping is used in the phase vocoder framework [38] but the QML is the first application for the high-order PPS estimation.