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The stability analysis of the adaptive three-stage Kalman filter

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ABSTRACT

An adaptive three-stage Kalman filter, which can track the fault and unknown inputs, is proposed by extending the special linear models of fault and unknown inputs to two general linear models without requirement of matrices ranks. It can be used when the fault and unknown inputs are not perfectly known. In this paper, the detailed stability analysis of the adaptive three-stage Kalman filter, which is applied for state and fault estimation of linear systems in the presence of unknown inputs, is investigated. Furthermore, an illustrative example is given to apply this filter and the performances of this filter are also verified by simulation.

1. Introduction

In a number of practical situations, such as model-based fault detection and isolation problems, and fault tolerant control problems [1–3], system models contain fault and unknown inputs. As a result of that, those problems have gained the interest of many researchers during the last decades. There are many methods when the true model of unknown inputs is available. Friedland's filter [4] has been proposed. But it is only optimal for constant bias. Then a series of optimal two-stage Kalman filters [5,6], which consider not only constant bias but also random bias, have been proposed. Recently, optimal three-stage Kalman filter (OThSKF) [7] has been proposed when the true dynamical evolutions of fault and unknown inputs are available. Furthermore, the OThSKF can be transformed to the optimal two-stage Kalman filter when the corresponding coefficients of models of the fault and unknown inputs are zero. However, there are some methods applied in the systems for the unavailable true model of unknown inputs. Robust two-stage Kalman filter (RTSF) and extension of the robust two-stage Kalman filter (ERTSF) have been proposed by Hsieh [8,9]. Ref. [10] has developed an adaptive two-stage Kalman filter and its stability has been analyzed in [11]. The same author of [7] has developed a robust three-stage Kalman filter (RThSKF) in the cases that the fault and unknown inputs are not perfectly known. Moreover, the RThSKF can be degraded to the RTSF and ERTSF. However, the RThSKF does not perform well when the model coefficients of the fault and unknown inputs are incomplete. What is worse, the RThSKF has many matrices ranks limits. Those will seriously affect the application of the RThSKF in engineering. As a result of that, an adaptive three-stage Kalman filter (AThSKF) is proposed to overcome those problems in this paper. For simplicity, here only simply summarize the corresponding main structure. Based on that, its detailed stability, which is the main work in this paper, is analyzed.

The remainder of this paper is organized as follows. Section 2 summarizes the AThSKF. In Section 3, we derive two Theorems. One is that the augmented state adaptive Kalman filter (AKF) is equivalent to the AThSKF. Therefore, the stability

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of the augmented state AKF can lead to the result that the AThSKF is stable. The other is the stability of the augmented state AKF, and based on that, the stability of the AThSKF is verified. In Section 4, the proposed filter is verified through an illustrative example. Conclusions are given in the last Section.

2. Adaptive three-stage Kalman filter

Consider the following linear discrete-time stochastic system represented by:

$$\mathbf{x}_{k+1}=\mathbf{A}_k^x x_k+\mathbf{B}_k\mathbf{u}_k+\mathbf{F}_k^x\mathbf{f}_k+\mathbf{E}_k^x\mathbf{d}_k+\mathbf{w}_k^x \tag{1.a}$$

$$\mathbf{f}_{k+1}=\mathbf{A}_k^f\mathbf{f}_k+\mathbf{w}_k^f \tag{1.b}$$

$$\mathbf{d}_{k+1}=\mathbf{A}_k^d\mathbf{d}_k+\mathbf{w}_k^d \tag{1.c}$$

$$\mathbf{z}_k=\mathbf{H}_k\mathbf{x}_k+\mathbf{F}_k^z\mathbf{f}_k+\mathbf{E}_k^z\mathbf{d}_k+\mathbf{v}_k \tag{1.d}$$

where $\mathbf{x}_k$ is the $n\times 1$ state vector.$\mathbf{u}_k$ is the $r\times 1$ known control input vector. $\mathbf{f}_k$ is the $p\times 1$ additive fault vector that occur in the system, for example measured angular rate fault and acceleration measurements fault in the deep space exploration field. $\mathbf{d}_k$ is the $q\times 1$ unknown inputs vector, for example atmosphere density uncertainty in Mars exploration. Furthermore, the models of fault and unknown inputs based on the past research are only the prior empirical models different from the true models.$\mathbf{z}_k$ is the $m\times 1$ measurement vector. All transmission matrices about $\mathbf{A}_k^x$, $\mathbf{B}_k$, $\mathbf{F}_k^x$, $\mathbf{E}_k^x$, $\mathbf{A}_k^f$, $\mathbf{A}_k^d$, $\mathbf{H}_k$, $\mathbf{F}_k^z$ and $\mathbf{E}_k^z$ have the appropriate dimensions. The noise sequences $\mathbf{w}_k^x$, $\mathbf{w}_k^f$, $\mathbf{w}_k^d$ and $\mathbf{v}_k$ are zero mean uncorrelated Gaussian random sequences with

$$E\left[\begin{bmatrix}\mathbf{w}_k^x\\ \mathbf{w}_k^d\\ \mathbf{w}_k^f\\ \mathbf{v}_k\end{bmatrix}\begin{bmatrix}\mathbf{w}_l^x\\ \mathbf{w}_l^d\\ \mathbf{w}_l^f\\ \mathbf{v}_l\end{bmatrix}^T\right]=\begin{bmatrix}\mathbf{Q}_k^x & 0 & 0 & 0\\ 0 & \mathbf{Q}_k^d & 0 & 0\\ 0 & 0 & \mathbf{Q}_k^f & 0\\ 0 & 0 & 0 & \mathbf{R}_k\end{bmatrix}\delta_{kl} \tag{1.e}$$

where δ_{kl} is the Kronecker delta, $\delta_{kl}=1$ if $k=l$, or $\delta_{kl}=0$.

The initial state $\mathbf{x}_0$, initial fault $\mathbf{f}_0$ and unknown inputs $\mathbf{d}_0$ satisfy the followings:

$E[\mathbf{x}_0]=\hat{\mathbf{x}}_0$ and $E[(\mathbf{x}_0-\hat{\mathbf{x}}_0)(\mathbf{x}_0-\hat{\mathbf{x}}_0)^T]=\mathbf{P}_0^x$

$$\begin{cases}E[\mathbf{f}_0]=\hat{\mathbf{f}}_0\\ E[\mathbf{d}_0]=\hat{\mathbf{d}}_0\\ E[(\mathbf{f}_0-\hat{\mathbf{f}}_0)(\mathbf{f}_0-\hat{\mathbf{f}}_0)^T]=\mathbf{P}_0^f\\ E[(\mathbf{d}_0-\hat{\mathbf{d}}_0)(\mathbf{d}_0-\hat{\mathbf{d}}_0)^T]=\mathbf{P}_0^d\\ E[(\mathbf{x}_0-\hat{\mathbf{x}}_0)(\mathbf{f}_0-\hat{\mathbf{f}}_0)^T]=\mathbf{P}_0^{xf}\\ E[(\mathbf{x}_0-\hat{\mathbf{x}}_0)(\mathbf{d}_0-\hat{\mathbf{d}}_0)^T]=\mathbf{P}_0^{xd}\\ E[(\mathbf{f}_0-\hat{\mathbf{f}}_0)(\mathbf{d}_0-\hat{\mathbf{d}}_0)^T]=\mathbf{P}_0^{fd}\end{cases} \tag{1.f}$$

Under this linear discrete-time stochastic system, the AThSKF, which uses three-stage U–V transformation in Ref. [7] and adaptive forgetting factor obtained from innovation information in Ref. [10], is presented in the following Definition 2.1.

Definition 2.1. The AThSKF is proposed, however, the detailed stability analysis of the AThSKF is the main work in this paper. So the recursive process using three-stage U–V transformation and adaptive forgetting factor technique is neglected here. In this Definition, only the AThSKF's main structure is summarized by the following four parts when the coefficients of models of the fault and unknown inputs given by (1.b) and (1.c) are not perfectly known:

The first part is the correction of the state and fault estimations

$$\begin{aligned}&\hat{\mathbf{x}}_k(-)=\overline{\mathbf{x}}_k(-)+\mathbf{U}_k^{12}\overline{\mathbf{f}}_k(-)+\mathbf{U}_k^{13}\overline{\mathbf{d}}_k(-),\quad \hat{\mathbf{P}}_k^{x*}(-)=\overline{\mathbf{P}}_k^{x*}(-)+\mathbf{U}_k^{12}\overline{\mathbf{P}}_k^{f*}(-)\mathbf{U}_k^{12T}+\mathbf{U}_k^{13}\overline{\mathbf{P}}_k^{d*}(-)\mathbf{U}_k^{13T}\\ &\hat{\mathbf{x}}_k(+)=\overline{\mathbf{x}}_k(+)+\mathbf{V}_k^{12}\overline{\mathbf{f}}_k(+)+\mathbf{V}_k^{13}\overline{\mathbf{d}}_k(+),\quad \hat{\mathbf{P}}_k^{x*}(+)=\overline{\mathbf{P}}_k^{x*}(+)+\mathbf{V}_k^{12}\overline{\mathbf{P}}_k^{f*}(+)\mathbf{V}_k^{12T}+\mathbf{V}_k^{13}\overline{\mathbf{P}}_k^{d*}(+)\mathbf{V}_k^{13T}\\ &\hat{\mathbf{f}}_k(-)=\overline{\mathbf{f}}_k(-)+\mathbf{U}_k^{23}\overline{\mathbf{d}}_k(-),\quad \hat{\mathbf{P}}_k^{f*}(-)=\overline{\mathbf{P}}_k^{f*}(-)+\mathbf{U}_k^{23}\overline{\mathbf{P}}_k^{d*}(-)\mathbf{U}_k^{23T}\\ &\hat{\mathbf{f}}_k(+)=\overline{\mathbf{f}}_k(+)+\mathbf{V}_k^{23}\overline{\mathbf{d}}_k(+),\quad \hat{\mathbf{P}}_k^{f*}(+)=\overline{\mathbf{P}}_k^{f*}(+)+\mathbf{V}_k^{23}\overline{\mathbf{P}}_k^{d*}(+)\mathbf{V}_k^{23T}\end{aligned} \tag{2}$$

The second part is the state subfilter

$$\overline{\mathbf{x}}_k(-)=\mathbf{A}_{k-1}^x\overline{\mathbf{x}}_{k-1}(+)+\mathbf{B}_{k-1}\mathbf{u}_{k-1}+\overline{\mathbf{u}}_{k-1}^x \tag{3.a}$$

$$\overline{\mathbf{P}}_k^{x*}(-)=\lambda_k^x[\mathbf{A}_{k-1}^x\overline{\mathbf{P}}_{k-1}^{x*}(+)\mathbf{A}_{k-1}^{xT}+\overline{\mathbf{Q}}_{k-1}^x] \tag{3.b}$$